Problem 4

(a) Let $y = \cos \pi x$.

(ii) The slope of PQ is

$$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.4\pi)}{0.5 - 0.4} = \frac{-\cos(0.4\pi)}{0.1} = -3.090170.$$

(iii) The slope of PQ is

$$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.49\pi)}{0.5 - 0.49} = -\frac{\cos(0.49\pi)}{0.01} = -3.141076.$$

(iv) The slope of PQ is

$$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.499\pi)}{0.5 - 0.499} = -\frac{\cos(0.499\pi)}{0.001} = -3.141586.$$

(vi) The slope of PQ is

$$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.6\pi)}{0.5 - 0.6} = \frac{\cos(0.6\pi)}{0.1} = -3.090170.$$

(vii) The slope of PQ is

$$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.51\pi)}{0.5 - 0.51} = \frac{\cos(0.51\pi)}{0.01} = -3.141076$$

(viii) The slope of PQ is

$$m_{PQ} = \frac{\cos(0.5\pi) - \cos(0.501\pi)}{0.5 - 0.501} = \frac{\cos(0.501\pi)}{0.001} = -3.141586.$$

- (b) The slope would be $-\pi$.
- (c) The equation of a line with slope $-\pi$ is $y y_0 = -\pi(x x_0)$ where the line passes through the point (x_0, y_0) . Therefore, since (0.5, 0) is on the line, we get

$$y = -\pi(x - 0.5) = -\pi x + \pi/2.$$

(d) Plot using Desmos.

Problem 8

(a) (i)
$$v_{ave} = \frac{s(2)-s(1)}{2-1} = 3 - (-3) = 6 \text{ cm/s.}$$

(ii)
$$v_{ave} = \frac{s(1.1) - s(1)}{1.1 - 1} \approx -4.7120 \,\mathrm{cm/s}.$$

- (iii) $v_{ave} = \frac{s(1.01) s(1)}{1.01 1} \approx -6.1341 \,\mathrm{cm/s}.$
- (iv) $v_{ave} = \frac{s(1.001) s(1)}{1.001 1} \approx -6.2683 \,\mathrm{cm/s}.$
- (b) We first give an estimation using a point on the left side of 1, say 0.999. We get $v_{ave} \approx -6.2746$ cm/s. So we estimate the instantaneous velocity as

$$v \approx \frac{-6.2683 + (-6.2746)}{2} = -6.2714 \,\mathrm{cm/s}.$$