Problem 4

(a) $\lim_{x\to 2^-} f(x) = 3$.

(b) $\lim_{x\to 2^+} f(x) = 1$.

(c) $\lim_{x\to 2}$ doesn't exist because the limit on the left is different from the limit on the right.

(d) $f(2) = 3$.

(e) $\lim_{x\to 4} f(x) = 4$ because the limit from the left and the limit from the right are 4.

(f) $f(4)$ doesn't exist.

Problem 16

Here is a graph that satisfies the requirementss.

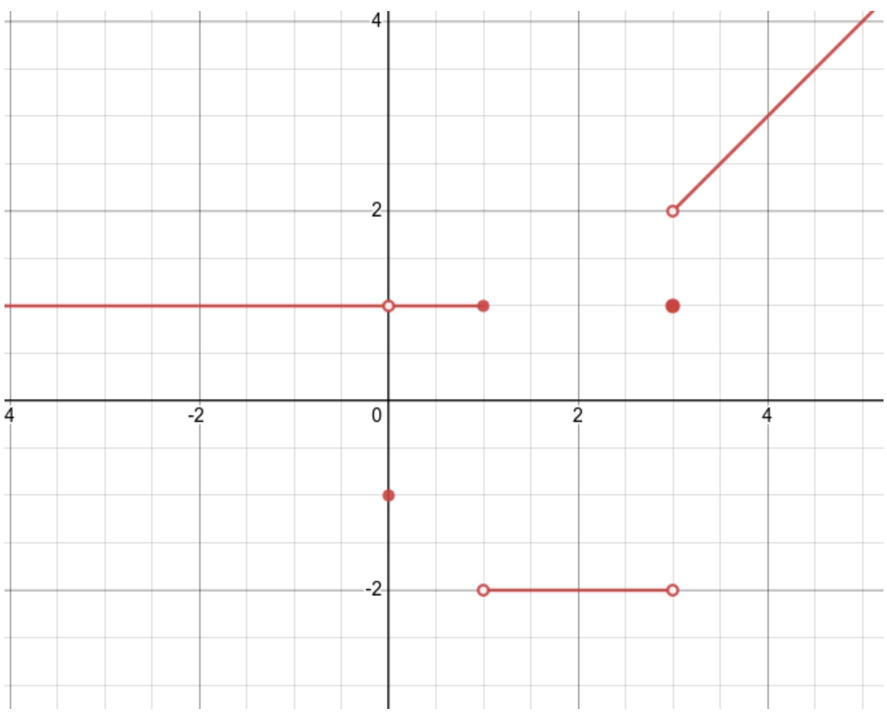

Figure 1: Graph of the function f

We see from the graph that

- $f(0) = -1$ and $f(3) = 1$;
- $\lim_{x\to 0} f(x) = 1$;

- $\lim_{x\to 3^-} f(x) = -2$;
- $\lim_{x\to 3^+} f(x) = 2$.

Problem 22

We build a table

x	$f(x)$
0.500000	131.312500
0.100000	88.410100
0.010000	80.804010
0.001000	80.080040
0.000100	80.008000
-0.500000	48.812500
-0.100000	72.390100
-0.010000	79.203990
-0.001000	79.920040
-0.000100	79.992000

We can guess from the values from the table that

$$\lim_{h\to 0} \frac{(2+h)^5 - 32}{h} = 80.$$

Problem 30

We see that the limit on the numerator is 6 and the limit of the denominator is 0^- meaning that the value approaches 0 from the left. So we have a number divided by a very small negative quantity. We then get

$$\lim_{x\to 5^-} \frac{x+1}{x-5} = 6/0^- = -\infty.$$

Problem 34

As x approaches 0^- (so x approaches 0 from the left), then $x-1$ approaches -1^- (so a number closed to -1 from the left) and $x+2$ approaches 2^- (so a number closed to 2 from the left). Therefore, the quotient $(x-1)/(x^2(x+2))$ approaches $-1^-/(0^-)^2(2^-) = -\infty$ because we divided by the square of a small negative number (really really close to zero, but negative) which turns out to be a small positive number.

As x approaches 0^+ (so x approaches 0 from the right), then $x-1$ approaches -1^+ (so a number closed to -1 from the right) and $x+2$ approaches 2^+ (so a number closed to 2 from the left). Therefore, the quotient $(x-1)/(x^2(x+2))$ approaches $-1^+/(0^+2^+) = -\infty$ because we divided by the square of a small positive number (really really close to zero, but positive) which turns out to be a small positive number.

Therefore, from the above observations, we conclude that

$$\lim_{x\to 0^-} \frac{x-1}{x^2(x+2)} = \lim_{x\to 0^+} \frac{x-1}{x^2(x+2)} = -\infty.$$

Problem 38

First, we notice that

$$\frac{x^2-2x}{x^2-4x+4} = \frac{x(x-2)}{(x-2)^2} = \frac{x}{(x-2)}.$$

Therefore, we see that, as x approaches 2 from the left, $x-2$ will be a small negative number (approaching 0 from the left) and x will approach 2. Therefore,

$$\lim_{x\to 2^-} \frac{x}{x-2} = \frac{2}{0^-} = -\infty.$$