Problem 8

Let's call L the limit. We have

$$\begin{aligned}
L &= \left(\lim_{t\to 2} \frac{t^2-2}{t^3-3t+5}\right)^2 && \text{(Power Rule)}\\
&= \left(\frac{\lim_{t\to 2} t^2-2}{\lim_{t\to 2} t^3-3t+5}\right)^2 && \text{(Quotient Rule)}\\
&= \left(\frac{\lim_{t\to 2} t^2 - \lim_{t\to 2} 2}{\lim_{t\to 2} t^3 - \lim_{t\to 2} 3t + \lim_{t\to 2} 5}\right)^2 && \text{(Sum \& Difference Rules)}\\
&= \left(\frac{(\lim_{t\to 2} t)^2 - \lim_{t\to 2} 2}{(\lim_{t\to 2} t)^3 - 3\lim_{t\to 2} t + \lim_{t\to 2} 5}\right)^2 && \text{(Product \& Power rules)}\\
&= \left(\frac{2^2-2}{2^3-6+5}\right)^2 = \frac{4}{49}.
\end{aligned}$$

So the limit is $L = 4/49$.

Problem 12

Unfortunately, we can't use the quotient rule because we have an indetermination $0/0$. Therefore, we have to check if we can rewrite the expression in the limit into another way so we can use the limit rules.

For $x \neq -3$, we have

$$\frac{x^2+3x}{x^2-x-12} = \frac{x(x+3)}{(x-4)(x+3)} = \frac{x}{x-4}.$$

Since $\lim_{x\to -3} x-4 = -7 \neq 0$, we can use the quotient rule and get

$$\lim_{x\to -3} \frac{x^2+3x}{x^2-x-12} = \lim_{x\to -3} \frac{x}{x-4} = \frac{\lim_{x\to -3} x}{\lim_{x\to -3} x-4} = \frac{3}{7}.$$

Problem 22

If we would use the quotient rule, we would get $0/0$. Since this is undefined, we most remove this undetermination.

For $x \neq 2$, we have

$$\frac{\sqrt{4u+1}-3}{u-2} = \left(\frac{\sqrt{4u+1}-3}{u-2}\right)\left(\frac{\sqrt{4u+1}+3}{\sqrt{4u+1}+3}\right) = \frac{4u+1-9}{(u-2)(\sqrt{4u+1}+3} = 4\frac{u-2}{(u-2)(\sqrt{4u+1}+3)}$$

and simplifying $u-2$, we obtain

$$\frac{\sqrt{4u+1}-3}{u-2} = \frac{4}{\sqrt{4u+1}+3}$$

Now, using the power rule and the sum rule, we see that

$$\lim_{u\to 2}\sqrt{4u+1}+3 = \sqrt{\lim_{u\to 2} 4u+1}+3 = \sqrt{8+1}+3 = 6.$$

Since 6 is different from zero, we can use the quotient rule! We therefore obtain

$$\lim_{u\to 2}\frac{\sqrt{4u+1}-3}{u-2} = \lim_{u\to 2}\frac{4}{\sqrt{4u+1}+3} = \frac{\lim_{u\to 2} 4}{\lim_{u\to 4}\sqrt{4u+1}+3} = \frac{4}{6} = \frac{2}{3}.$$

Problem 26

We have

$$\frac{1}{t} - \frac{1}{t^2+t} = \frac{1}{t} - \frac{1}{(t+1)t} = \frac{t+1-1}{t(t+1)} = \frac{1}{t+1}.$$

So the limit is

$$\lim_{t\to 0}\left(\frac{1}{t} - \frac{1}{t^2+t}\right) = \lim_{t\to 0}\frac{1}{1+t} = 1.$$

Problem 36

Since $-1 \le \sin A \le 1$ for any real number A, we know that $-1 \le \sin\left(\frac{\pi}{x}\right) \le 1$. Therefore, multiplying by $\sqrt{x^3+x^2}$, we obtain

$$-\sqrt{x^3+x^2} \le \sqrt{x^3+x^2}\sin\left(\frac{\pi}{x}\right) \le \sqrt{x^3+x^2}.$$

Using the power rule and the sum rule, we have

$$\lim_{x\to 0}\sqrt{x^3+x^2} = 0 = \lim_{x\to 0} -\sqrt{x^3+x^2}.$$

Therefore, by the Squeeze Theorem, we can conclude that

$$\lim_{x\to 0}\sqrt{x^3+x^2}\sin\left(\frac{\pi}{x}\right) = 0.$$

Problem 42

We have to check if the limit from the left is the same as the limit from the right.

For the limit from the left, we will approach -6 with numbers x less than -6. Therefore, $x+6<0$ for $x<-6$ and $|x+6| = -(x+6) = -x-6$. Therefore, we get

$$\lim_{x\to-6^-} \frac{2x+12}{|x+6|} = \lim_{x\to-6^-} 2\frac{x+6}{-(x+6)} = \lim_{x\to-6^-} -2 = -2.$$

For the limit from the right, we will approach -6 with numbers x greater than -6. This means $x+6>0$ (when $x>-6$) and therefore $|x+6| = x+6$. We then get

$$\lim_{x\to-6^+} 2\frac{x+6}{|x+6|} = \lim_{x\to-6^+} 2\frac{x+6}{x+6} = \lim_{x\to-6^+} 2 = 2.$$

The limit from the left is -2 and the limit from the right is 2. Since they are different, we conclude that the limit

$$\lim_{x\to-6} \frac{2x+12}{|x+6|}$$

does not exist.

Problem 60

a) Since $\lim_{x\to0} x^2$ exists and $\lim_{x\to0} f(x)/x^2$ also exists, from the properties of the limits, we have

$$\left(\lim_{x\to0} x^2\right)\left(\lim_{x\to0} \frac{f(x)}{x^2}\right) = \lim_{x\to0} x^2\left(\frac{f(x)}{x^2}\right) = \lim_{x\to0} f(x).$$

But $\lim_{x\to0} x^2 = 0$ and $\lim_{x\to0} f(x)/x^2 = 5$, we get $\lim_{x\to0} f(x) = 0\times5 = 0$.

b) We use the same strategy. Since $\lim_{x\to0} x$ exists and $\lim_{x\to0} f(x)/x^2$ also exists, we get

$$\left(\lim_{x\to0} x\right)\left(\lim_{x\to0} \frac{f(x)}{x^2}\right) = \lim_{x\to0} x\left(\frac{f(x)}{x^2}\right) = \lim_{x\to0} \frac{f(x)}{x}.$$

But $\lim_{x\to0} x = 0$ and $\lim_{x\to0} f(x)/x = 5$, we get $\lim_{x\to0} f(x)/x = 0\times5 = 0$.