Problem 18

As $x \to -2^-$, we have $f(x) \to -\infty$ and as $x \to -2^+$, we have $f(x) \to \infty$. So we have an infinite discontinuity.

Problem 36

The function $x + \sin x$ is continuous because it is the sum of two continuous functions. Now, $\sin x$ is continuous and therefore the composition $\sin(x + \sin x)$ is continuous. In particular, the function $x \mapsto \sin(x + \sin x)$ is continuous at $x = \pi$. Using the continuity, this means that

$$\lim_{x \to \pi} \sin(x + \sin x) = \sin(\pi + \sin \pi) = \sin(\pi + 0) = 0.$$

Problem 56

Let $f(x) = \sin x - x^2 + x$. We have $a = 1$ and $b = 2$.

We will verify if the hypothesis of the Intermediate Value Theorem are verified.

- The function f is a sum of continuous function on all of $(-\infty, \infty)$, therefore f is continuous on all of $(-\infty, \infty)$. In particular, the function f is continuous on $(1, 2)$.
- $f(1) = \sin(1) - 1^2 + 1 = \sin(1) > 0$ because for any $0 < x < \pi$, we have $\sin(x) > 0$.
- $f(2) = \sin(2) - 4 + 2 = \sin(2) - 2 < 0$ because $\sin(1) < 1 < 2$.

All the hypothesis of the IVP are satisfied. We therefore conclude that there is some c, between 1 and 2, such that $f(c) = 0$. This means that

$$\sin(c) - c^2 + c = 0 \quad \iff \quad \sin(c) = c^2 - c$$

for some c such that $1 < c < 2$.

Problem 58 (a)

We have $f(0) = 3$ and $f(-1) = -1 - 1 - 2 + 3 = -1$. So, we have $f(-1) < 0$ and $f(0) > 0$. So, by the intermediate Theorem, with $N = 0$, there is a number $c \in (-1, 0)$ such that $f(c) = 0$.