Problem 12

That $t = 0$, the slope of the tangent line is positive and quite small. When we move towards $t = 5$, the slope increases and attain a maximum around $t = 6$. Then the slope decreases as we more towards $t = 10$. The slope becomes really small (close to zero) when we reach $t = 15$. The graph show look like this:

Problem 20

Using the definition, we have

$$\begin{aligned} f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} &= \lim_{h\to 0} \frac{m(x+h) + b - (mx+b)}{h} \\ &= \lim_{h\to 0} \frac{mx + mh + b - mx - b}{h} \\ &= \lim_{h\to 0} \frac{mh}{h} = m. \end{aligned}$$

Therefore, $f'(x)$ exists for any x and $f'(x) = m$.

Problem 25

The domain of the function is $(-\infty, 9]$. The derivative at x is

$$\begin{aligned} f'(x) = \lim_{h\to 0} \frac{\sqrt{9-x-h} - \sqrt{9-x}}{h} &= \lim_{h\to 0} \frac{9-x-h-9+x}{h(\sqrt{9-x-h} + \sqrt{9-x}} \\ &= \lim_{h\to 0} -\frac{1}{\sqrt{9-x-h} + \sqrt{9-x}} \\ &= -\frac{1}{2\sqrt{9-x}}. \end{aligned}$$

So $f'(x) = -1/2\sqrt{9-x}$ and the domain of f' is $(-\infty, 9)$.

Problem 26

By definition, we have

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}.$$

Let's simplify the difference quotient:

$$
\begin{aligned}
\frac{f(x+h)-f(x)}{h} &= \frac{\frac{(x+h)^2-1}{2(x+h)-3} - \frac{x^2-1}{2x-3}}{h(2x-3)(2x+2h-3)} \\
&= \frac{(x^2+2xh+h^2-1)(2x-3)-(x^2-1)(2x+2h-3)}{h(2x-3)(2x+2h-3)} \\
&= \frac{(x^2-1)(2x-3)+(2xh+h^2)(2x-3)-(x^2-1)(2x-3)-2h(x^2-1)}{h(2x-3)(2x+2h-3)} \\
&= \frac{4x^2h-6xh+2xh^2-3h^2-2x^2h+2h}{h(2x-3)(2x+2h-3)} \\
&= \frac{2x^2h+2xh^2-6xh-3h^2+2h}{h(2x-3)(2x+2h-3)} \\
&= \frac{2x^2+2xh-6x-3h+2}{(2x-3)(2x+2h-3)}.
\end{aligned}
$$

Now we just have to take the limit as $h \to 0$:

$$
f'(x) = \lim_{h\to 0} \frac{2x^2+2xh-6x-3h+2}{(2x-3)(2x+2h-3)} = \frac{2x^2-6x+2}{(2x-3)^2}.
$$

Problem 28

The domain of the function is $[0,\infty)$. Using the definition, we have

$$
\begin{aligned}
f'(x) = \lim_{h\to 0}\frac{f(x+h)-f(x)}{h} &= \lim_{h\to 0}\frac{(x+h)^{3/2}-x^{3/2}}{h} \\
&= \lim_{h\to 0}\left(\frac{(x+h)^{3/2}-x^{3/2}}{h}\right)\left(\frac{(x+h)^{3/2}+x^{3/2}}{(x+h)^{3/2}+x^{3/2}}\right) \\
&= \lim_{h\to 0}\frac{(x+h)^3-x^3}{h((x+h)^{3/2}+x^{3/2})} \\
&= \lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h((x+h)^{3/2}+x^{3/2})} \\
&= \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h((x+h)^{3/2}+x^{3/2})} \\
&= \lim_{h\to 0}\frac{3x^2+3xh+h^2}{(x+h)^{3/2}+x^{3/2}} \\
&= \frac{3x^2}{2x^{3/2}} \\
&= \frac{3}{2}x^{1/2}.
\end{aligned}
$$

Therefore, we get $f'(x) = (3/2)x^{1/2}$. The domain of the derivative is $[0,\infty)$.

Problem 32

(a) By definition, we have

$$\begin{aligned}
f'(x) &= \lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\\
&= \lim_{h\to 0}\frac{x+h+\frac{1}{x+h}-x-\frac{1}{x}}{h}\\
&= \lim_{h\to 0}\frac{h+\frac{1}{x+h}-\frac{1}{x}}{h}\\
&= \lim_{h\to 0}\frac{(x+h)xh+x-x-h}{(x+h)xh}\\
&= \lim_{h\to 0}\frac{x^2h+xh^2-h}{(x+h)xh}\\
&= \lim_{h\to 0}\frac{x^2+xh-1}{(x+h)x}
\end{aligned}$$

Then use the Quotient Rule to evaluate the last limit. We get

$$f'(x) = \frac{x^2-1}{x^2} = 1-\frac{1}{x^2}.$$

The domain of f' is $(-\infty,0)\cup(0,\infty)$.

(b) Here are the graphs of f and f'. Desmos was used to draw the figure.

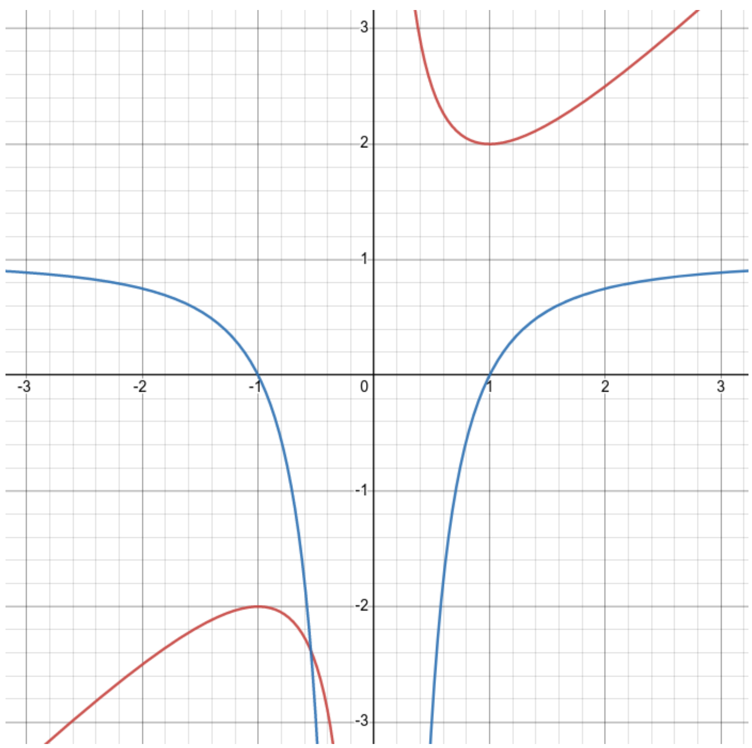

Figure 1: In red, graph of $f(x)$ and, in blue, the graph of $f'(x)$

Problem 40

The function is not differentiable at $x=-1$ because f is not continuous.

The function is not differentiable at $x = 2$ because there is a corner in the graph of f (the limit slope from the left and from the right are not the same).

Problem 48

We have to take a look at the slope of the tangent lines in each graph.

We can see that the curve "c" is positive where the slopes of the tangents to the graph of the curve "d" are postive. Also, we see that the curve "c" is negative when the slopes of the tangents to the graph of the curve "d" are negative. So the curve "c" represents the derivative of "d".

We remark that the sign of the y-coordinate of the points of the curve "b" is the same as the slopes of the tangents to the curve "c". So curve "b" is the derivative of the curve "c".

Finally, we see that the sign of the slopes of the tangents to the curve "b" are always positive or zero and this is the same sign as the y-coordinate of the points on the curve "a". So the curve "a" is the derivative of the curve "b".

In summary, we have

$$f \leftrightarrow d,\ f' \leftrightarrow c,\ f'' \leftrightarrow b \text{ and } f''' \leftrightarrow a.$$