

Problem 1

Using the product rule, we have

$$f'(x) = (x^2)' \sin x + x^2(\sin x)' = 2x \sin x + x^2 \cos(x).$$

Problem 3

From the sum and difference rules, we have

$$f'(x) = 3(\cot x)' - 2(\cos x)' = -3 \csc^2(x) + 2 \sin x.$$

Notice that the negative sign became a plus sign because the derivative of $\cos x$ is $-\sin x$.

Problem 5

There are two ways to complete the problem.

1. We can use the product rule:

$$\begin{aligned} \frac{dy}{d\theta} &= \frac{d}{d\theta}(\sec \theta) \tan \theta + \sec \theta \frac{d}{d\theta}(\tan \theta) \\ &= \sec \theta \tan^2 \theta + \sec^3 \theta. \end{aligned}$$

2. We can rewrite the expression of y as

$$y = \frac{1}{\cos \theta} \frac{\sin \theta}{\cos \theta} = \frac{\sin \theta}{\cos^2 \theta}.$$

We then use the quotient rule:

$$\begin{aligned} y' &= \frac{\frac{d}{d\theta}(\sin \theta) \cos^2 \theta - \sin \theta \frac{d}{d\theta}(\cos^2 \theta)}{\cos^4 \theta} \\ &= \frac{\cos^3 \theta + 2 \sin^2 \theta \cos \theta}{\cos^4 \theta} \\ &= \frac{\cos^2 \theta + 2 \sin^2 \theta}{\cos^3 \theta}. \end{aligned}$$

We can simplify further using $\sin^2 \theta + \cos^2 \theta = 1$ and then obtain

$$y' = \frac{\cos^2 + \sin^2 \theta + \sin^2 \theta}{\cos^3 \theta} = \frac{1 + \sin^2 \theta}{\cos^3 \theta}.$$

Problem 9

Using the quotient rule, we have

$$\begin{aligned}\frac{dy}{dx} &= \frac{1(2 - \tan x) - x(-\sec^2 x)}{(2 - \tan x)^2} \\ &= \frac{2 + x \sec^2 x - \tan x}{(2 - \tan x)^2}.\end{aligned}$$

Problem 15

Using the product rule a first time:

$$f'(\theta) = (\theta)' \cos \theta \sin \theta + \theta(\cos \theta \sin \theta)'$$

Using the product rule a second time:

$$(\cos \theta \sin \theta)' = -\sin^2 \theta + \cos^2 \theta = \cos(2\theta).$$

Therefore, we get

$$\begin{aligned}f'(\theta) &= (1) \cos \theta \sin \theta + \theta \cos(2\theta) \\ &= \cos \theta \sin \theta + \theta \cos(2\theta) \\ &= \frac{1}{2} \sin(2\theta) + \theta \cos(2\theta).\end{aligned}$$