Problem 8

By the Chain Rule, we have

$$F'(x) = 99(1 + x + x^2)^{98}(1 + 2x) = 99(1 + 2x)(1 + x + x^2)^{98}.$$

Problem 10

We have, by the chain rule,

$$g'(x) = \frac{3}{2}(2 - \sin x)^{1/2}(-\cos x) = -\frac{3\cos x\sqrt{2 - \sin x}}{2}.$$

Problem 11

The outer function is $f(t) = \frac{1}{t^2}$ and the inside function is $g(t) = \cos t + \tan t$. This implies that

$$A(t) = f(g(t)).$$

Using the Chain Rule, we have

$$A'(t) = f'(g(t))g'(t).$$

Now, we have

$$f'(t) = (t^{-2})' = -2t^{-3} = -\frac{2}{t^3}.$$

Also, we have

$$g'(t) = -\sin t + \sec^2 t$$

and therefore

$$\begin{aligned} A'(t) &= -\frac{2}{(\cos t + \tan t)^3}(-\sin t + \sec^2 t) \\ &= -2\frac{\sec^2 t - \sin t}{(\cos t + \tan t)^3}. \end{aligned}$$

Problem 30

We use the chaine rule and we get

$$ds/dt = \frac{1}{2}\Big(\frac{1 + \sin t}{1 + \cos t}\Big)^{-1/2}\frac{d}{dx}\Big(\frac{1 + \sin t}{1 + \cos t}\Big).$$

The derivative of $(1+\sin t)/(1+\cos t)$ is

$$\frac{\cos t(1+\cos t)-(1+\sin t)(-\sin t)}{(1+\cos t)^2}=\frac{\cos t+\cos^2 t+\sin t+\sin^2 t}{(1+\cos t)^2}=\frac{1+\sin t+\cos t}{(1+\cos t)^2}.$$

Thus, the final answer looks like

$$ds/dt=\frac{1+\sin t+\cos t}{2(1+\cos t)^{3/2}\sqrt{1+\sin t}}.$$