Problem 12

We suppose that $y = f(x)$. Set $y' = dy/dx$. We differentiate with respect to x on each side of the equation:

$$-\sin(xy)(y + xy') = \cos(y)y'$$

and so

$$-y\sin(xy) - xy'\sin(xy) = y'\cos(y)$$

and then

$$y' = -\frac{y\sin(xy)}{x\sin(xy) + \cos(y)}.$$

Problem 32

We suppose that $y = f(x)$ and differentiate each side of the equation. We obtain

$$2yy'(y^2 - 4) + 2y^3y' = 2x(x^2 - 5) + 2x^3.$$

So, now we have to isolate y'. After distributing y and x, we obtain

$$y'(2y^3 - 8y + 2y^3) = 2x^3 - 10x + 2x^3.$$

We then find

$$y' = \frac{x(2x^2 - 5)}{2y(y^2 - 2)}$$

The equation of the tangent line is $y + 2 = m(x - 0)$ where $m = y'(0)$. So, replacing $x = 0$ and $y = -2$ in the above equation for y', we get $m = 0$. Thus, we obtain

$$y = -2.$$