## Problem 6

We denote by

- $V(t)$: volume of the sphere (in mm$^3$).
- $r(t)$: radius of the sphere (in mm).
- $t$: time in seconds.

We know that

$$\frac{dr}{dt} = 4\text{mm}/s.$$

The goal is to find

$$\left.\frac{dV}{dt}\right|_{r=40}.$$

The connection between $V$ and $r$ is

$$V = \frac{4}{3}\pi r^3.$$

Taking the derivative, we obtain

$$\frac{dV}{dt} = 4\pi r^2\Big(\frac{dr}{dt}\Big).$$

Therefore, replacing $r$ by 40, we get

$$\frac{dV}{dt} = 4\pi(40)^2 4 = 25600\pi\,\text{mm}^3.$$

## Problem 12

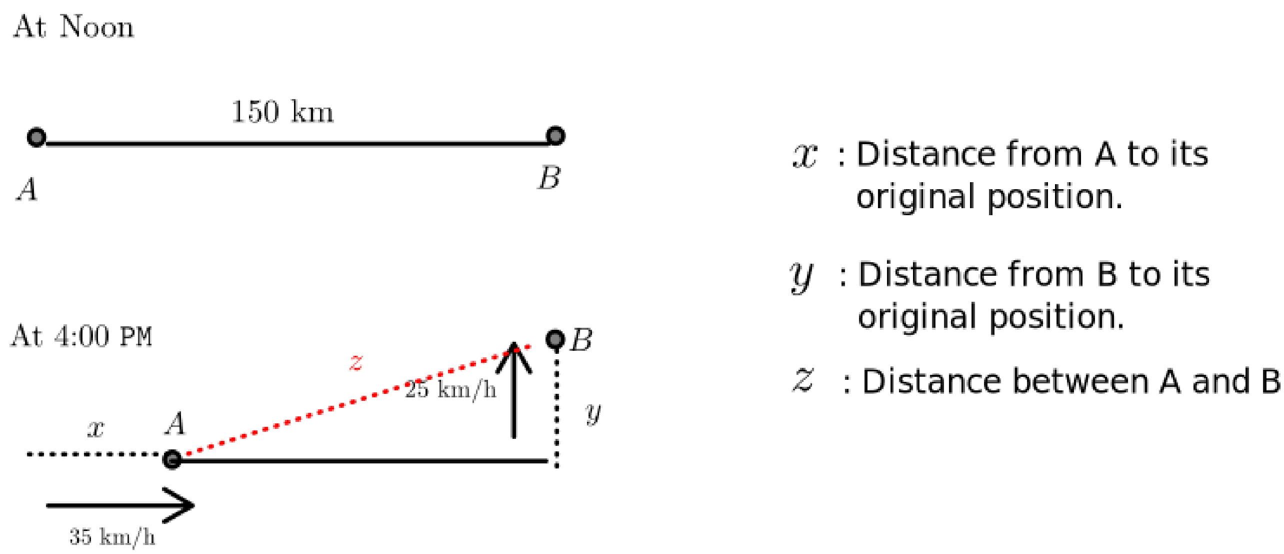
We differentiate with respect $t$ the equation in $x$ and $y$ to get

$$x'y + xy' = 0..$$

Replacing $x$ by 4, $y$ by 2, and $y' = -3$, we then obtain $x' = 6$ cm/s.

## Problem 16

First, let's draw a picture and introduce some notations. The known information is $dx/dt = 35$




and $dy/dt = 25$. What we would like to know is $dz/dt$.

The link between $x$, $y$ and $z$ is given by the pythagorean Theorem:

$$z^2 = (150 - x)^2 + y^2$$

where $150 - x$ is the distance from the boat $A$ to the original position of the boat $B$. Taking the derivative with respect to time gives

$$\begin{aligned} & 2z(dz/dt) = 2(150 - x)(-dx/dt) + 2y(dy/dt). \\ \iff \quad & dz/dt = ((150 - x)/z)(-dx/dt) + (y/z)(dy/dt). \end{aligned}$$

From noon to 4:00PM, the boat A travelled $4 \times 35 = 140$ km and the boat B travelled $4 \times 25 = 100$ km. So $x = 140$, $y = 100$, and $z = \sqrt{10^2 + 100^2} = 10\sqrt{101}$. Replacing everything in the last equations above, we obtain

$$dz/dt = (1/\sqrt{101})(-35) + (10/\sqrt{101})(25) = 215/\sqrt{101} \approx 25 \text{ km/h}.$$

Thus, $dz/dt \approx 25$ km/h.

## Problem 22

We denote by

- $x(t)$: the distance from the bow of the boat and the bottom of the dock (in meters).
- $z(t)$: the distance from the bow of the boat and the dock.
- $t$: time in seconds.

We know that

$$\frac{dz}{dt} = 1\text{m/s}.$$

The goal is to find

$$\left.\frac{dx}{dt}\right|_{x=8m}.$$

The connection between $z$ and $x$ is via the pythagorean theorem

$$x^2 + 1^2 = z^2 \quad \Rightarrow \quad x^2 + 1 = z^2.$$

Taking the derivative, we find that

$$2x\frac{dx}{dt} = 2z\frac{dz}{dt} \quad \Rightarrow \quad x\frac{dx}{dt} = z\frac{dz}{dt}.$$

With $x = 8$, we find that $z = \sqrt{1 + 8^2} = \sqrt{65}$. Therefore, pluging all the information in, we find

$$8\frac{dx}{dt} = \sqrt{65} \cdot 1 \quad \Rightarrow \quad \frac{dx}{dt} = \frac{\sqrt{65}}{8}\text{m/s}.$$