Problem 4

The linearization is given by

$$L(x) = f(3) + f'(3)(x-3).$$

We have

$$f'(x) = \frac{-2x}{(x^2-5)^{3/2}}.$$

Therefore, we have $f'(3) = \frac{-6}{8} = -\frac{3}{4}$. We also have $f(3) = 1$. Therefore, the linearization is

$$L(z) = 1 - \frac{3}{4}(x-3)$$

Problem 24

We see that

$$4.002 = 4 + 0.002.$$

Therefore, the value 0.002 will by my x in the linearization and suggest

$$f(x) = \frac{1}{x+4}.$$

We see that $f(0.002) = 1/4.002$. Therefore, a linear approximation of f around $x = 0$ will be useful to approximation $1/4.002$. We have

$$f'(x) = -\frac{1}{(x+4)^2} \quad \Rightarrow \quad f'(0) = -\frac{1}{16}.$$

Since $f(0) = 1/4$, we have

$$L(x) = 1/4 - \frac{x}{16}.$$

Using the linearization of f, we find that

$$\frac{1}{4.002} = f(0.002) \approx L(0.002) = 0.25 - \frac{0.002}{16} = 0.25 - 0.000125 = 0.249875.$$

Therefore, we have

$$\frac{1}{4.002} \approx 0.249875.$$