Problem 30

The derivative is $f'(x) = 3x^2 + 12x - 15$. So the critical numbers are the solutions to the equation $x^2 + 4x - 5 = 0$. The solutions are $x = 1$ and $x = -5$. So, the critical numbers are $x = 1$ and $x = -5$ (the derivative exists everywhere).

Problem 34

The function is not differentiable at $c = 4/3$ because g has a corner there. Therefore $c = 4/3$ is a critical point.

Now, for $t < 4/3$, we have that $3t - 4 < 0$ and

$$g(t) = -(3t - 4) = 4 - 3t.$$

We have $g'(t) = -3$ and the number -3 is never zero. So no critical point for $t < 4/3$.

Now, for $t > 4/3$, we have that $3t - 4 > 0$ and

$$g(t) = 3t - 4.$$

We have $g'(t) = 3$ and the number 3 is never zero. So, no critical point for $t > 4/3$.

In summary, there is only one critical number at $c = 4/3$.

Problem 38

The derivative of g is

$$g'(x) = -\frac{2}{3}\frac{x}{(4 - x^2)^{2/3}}.$$

The derivative does not exist when the denominator is zero. The denominator is zero if

$$4 - x^2 = 0 \iff x = \pm 2.$$

The derivative is zero if the numerator of g' is zero. The numerator is zero if $x = 0$.

Therefore, the critical points are $c = -2$, $c = 0$, and $c = 2$.

Problem 50

The derivative of the function is $f'(t) = 6t(t^2 - 4)^3$. So the critical points within $(-2, 3)$ are $t = 0$ and $t = 2$.

We have $f(-2) = 0$, $f(0) = -64$, $f(2) = 0$, and $f(3) = 125$. So the maximum is

$$M = 125$$

and the minimum is

$$m = -64.$$

Problem 52

The derivative of f is

$$f'(x) = \frac{(x^2 - x + 1) - x(2x - 1)}{(x^2 - x + 1)^2} = \frac{-x^2 + 1}{(x^2 - x + 1)^2} = -\frac{1 - x^2}{(x^2 - x + 1)^2}.$$

The critical points of f are

- When $f'(x)$ does not exists. The denominator is zero if

  $$x^2 - x + 1 = 0.$$

  The discrimant of this quadratic is

  $$b^2 - 4ac = 1 - 4 = -3.$$

  Since the discrimant is negative, the expression $x^2 - x + 1$ is never zero.

- When $f'(x)$ is zero. The derivative f' is zero if

  $$1 - x^2 = 0 \iff x = \pm 1.$$

The critical points of f are therefore $c = -1$ and $c = 1$.

Using the closed interval method, one critical point is within the interval $[0, 3]$. Therefore, we have

$$\max f(x) = \max\{f(0), f(1), f(3)\} = \max\{0, 1, 3/7\} = 1.$$

Problem 54

We have to find the critical points inside the interval $(0, 2)$. The derivative of f is

$$f'(t) = \frac{(1 + t^2)/2\sqrt{t} - \sqrt{t}(2t)}{(1 + t^2)^2} = \frac{1 + t^2 - 4t^2}{2\sqrt{t}(1 + t^2)^2} = \frac{1 - 3t^2}{2\sqrt{t}(1 + t^2)^2}.$$

The zeros of the derivative are at $t = \pm\sqrt{1/3}$. We have to discard $-\sqrt{1/3}$ because it's not in the interval. The derivative exists at every point in $(0, 2)$.

Now the maximum and the minimum will be given by the max of the values $f(0) = 0$, $f(\sqrt{1/3}) \approx 0.5698$, and $f(2) \approx 0.2828$. So the maximum

$$M = 0.5698$$

and the minimum

$$m = 0.2828.$$