Problem 6

We set $f(x) = 2x^3 - 3x^2 + 2$. The derivative of the function is

$$f'(x) = 6x^2 - 6x.$$

Newton's algorithm gives

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}.$$

We start we $x_1 = -1$. We have

$$f(x_1) = -3 \quad \text{and} \quad f'(x_1) = 12.$$

Therefore, we obtain

$$x_2 = -1 + \frac{3}{12} = -1 + \frac{1}{4} = -\frac{3}{4}.$$

We now use x_2 to obtain x_3. We have

$$f(x_2) = -0.53125 \quad \text{and} \quad f'(x_2) = 63/8.$$

Therefore, we obtain

$$x_3 = -\frac{3}{4} + \frac{0.53125}{63/8} \approx -0.6825$$

Problem 34

To find the maximum value, we will use the interval method. We have to find the critical numbers inside $(0, \pi)$. The derivative of f is

$$f'(x) = \cos x - x \sin x.$$

The derivative exists everywhere, so the critical numbers are the zero of f'. We will use Newton's method to find the zero of f'. If $x = 0$, then $f'(x) = 0$. But x is not inside the interval $(0, \pi)$. We will search for another zero inside $(0, \pi)$. The Newton's method tells us that the critical number c will be approximated by

$$x_{n-1} - f'(x_{n-1})/f''(x_{n-1})$$

where x_1 is an initial guess within $(0, \pi)$.

Let $x_1 = \pi/2$. We have $f''(x) = -2\sin x - x\cos x$. So

$$c \approx x_{n-1} - \frac{\cos x_{n-1} - x_{n-1}\sin(x_{n-1})}{-2\sin x_{n-1} - x_{n-1}\cos x_{n-1}}.$$

Apprying Newton's method several times, we get the following approximations of c:

Iteration	x_n
2	0.7853981633974483
3	0.8624434632122491
4	0.8603349794247831
5	0.8603335890199867
6	0.8603335890193797

We see that, after the fifth iteration, the first six digits are stable. So $c \approx 0.860333$.

Now, we have

$$\max f(x) = \max\{f(0), f(0.860333), f(\pi)\} = \max\{0, 0.561096, -3.141592\} = 0.561096.$$