Problem 2

The first term becomes $\frac{x^3}{3}$, the second term becomes $-\frac{3}{2}x^2$ and the last term becomes $2x$. Therefore, the most general antiderivative is

$$F(x) = \frac{x^3}{3} - \frac{3x^2}{2} + 2x + C.$$

Problem 6

We can see that

$$f(x) = (x-5)^2 = (x-5)^2 \frac{d}{dx}(x-5)$$

and so from the Chain Rule,

$$F(x) = \frac{(x-5)^3}{3} + C.$$

Problem 12

We turn $\sqrt[3]{x^2}$ and $\sqrt{x}$ into exponent and then $f(x) = x^{2/3} + xx^{1/2} = x^{2/3} + x^{3/2}$. Therefore,

$$F(x) = \frac{x^{2/3+1}}{2/3+1} + \frac{x^{3/2+1}}{3/2+1} + C = \frac{x^{5/3}}{5/3} + \frac{x^{5/2}}{5/2} + C = \frac{3}{5}x^{5/3} + \frac{2}{5}x^{5/2} + C.$$

Therefore, $F(x) = \frac{3}{5}x^{5/3} + \frac{2}{5}x^{5/2} + C$.

Problem 14

We simplify $g(x)$:

$$g(x) = \frac{5}{x^6} - \frac{4}{x^3} + 2 = 5x^{-6} - 4x^{-3} + 2.$$

Therefore, we obtain

$$G(x) = \frac{5}{-6+1}x^{-6+1} - \frac{4}{-3+1}x^{-3+1} + 2x + C = \frac{5}{-5}x^{-5} - \frac{4}{-2}x^{-2} + 2x + C.$$

After simplifications, $G(x) = -x^{-5} + 2x^{-2} + 2x + C$.

Problem 16

The antiderivative of $\cos t$ is $\sin t$ because $(sint)' = \cos t$ and the antiderivative of $\sin t$ is $-\cos(t)$ because $(-\cos t)' = -(-\sin t) = \sin t$. Therefore,

$$F(t) = 3\sin t + 4\cos t + C.$$

Problem 22

The most general antiderivative of $f(x)$ is

$$F(x) = \frac{x^2}{2} - 2\cos x + C.$$

Since $F(0) = -6$, we then have the following equation for C:

$$-6 = \frac{0^2}{2} - 2\cos(0) + C \iff -6 = -2 + C \iff -4 = C.$$

Therefore, the antiderivative satisfying $F(0) = -6$ is $F(x) = \frac{x^2}{2} - 2\cos x - 4$.

Problem 24

Let $g(x) = f'(x)$, so that $g'(x) = f''(x)$. We first find $g(x)$. We have $g'(x) = x^6 - 4x^4 + x + 1$, so that

$$g(x) = \frac{x^7}{7} - \frac{4x^5}{5} + \frac{x^2}{2} + x + C.$$

We know that $f'(x) = g(x)$, so that

$$f(x) = \frac{x^8}{56} - \frac{2x^6}{15} + \frac{x^3}{6} + \frac{x^2}{2} + Cx + D.$$

Problem 33

We simplify the expression of $f'(x)$:

$$f'(x) = \sec^2(t) + \sec t \tan t.$$

An antiderivative for $\sec^2(t)$ is $\tan(t)$ because $(\tan t)' = \sec^2 t$ and an antiderivative for $\sec t \tan t$ is $\sec t$ because $(\sec t)' = \sec t \tan t$. Therefore, we have

$$f(x) = \tan t + \sec t + C.$$

Since $f(\pi/4) = -1$, we then have the following equation for C:

$$-1 = \tan(\pi/4) + \sec(\pi/4) + C \iff -1 = 1 + \frac{1}{\sqrt{22}} + C \iff 2 - \frac{2}{\sqrt{2}} = C.$$

Therefore, $f(x) = \tan t + \sec t + 2 - \frac{2}{\sqrt{2}}$.