Problem 4

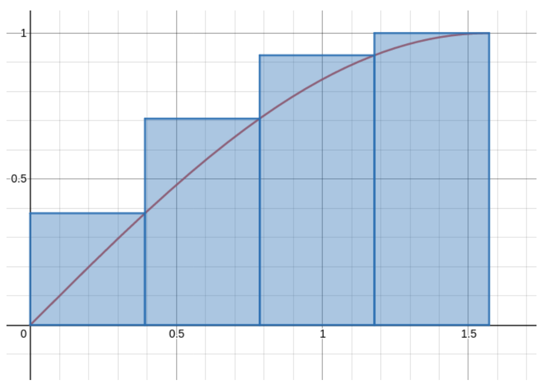
a) We have $n = 4$ and so $\Delta x = (\pi/2 - 0)/4 = \pi/8$. The sample points are

$$x_1 = \pi/8,\ x_2 = \pi/4,\ x_3 = 3\pi/8,\ x_4 = \pi/2.$$

So, we obtain

$$\begin{aligned} A &\approx \sin(\pi/8)\Delta x + \sin(\pi/4)\Delta x + \sin(3\pi/8)\Delta x + \sin(\pi/2)\Delta x \\ &= (\pi/8)(\sin(\pi/8) + \sin(\pi/4) + \sin(3\pi/8) + \sin(\pi/2)) \\ &\approx 1.18346. \end{aligned}$$

Here is the graph of the function and the approximate squares. We see that we overestimated the area under the curve.



b) We have $n = 4$ and so $\Delta x = (\pi/2 - 0)/4 = \pi/8$. The sample points are

$$x_1 = 0,\ x_2 = \pi/8,\ x_3 = \pi/4,\ x_4 = 3\pi/8.$$

We then obtain

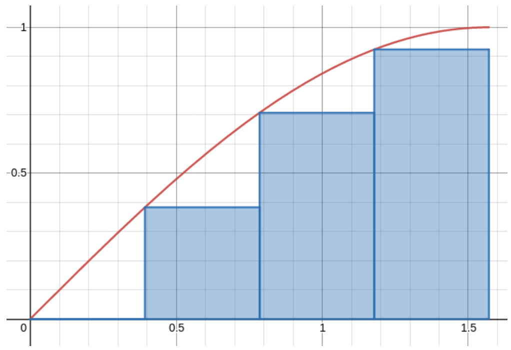
$$\begin{aligned} A &\approx \sin(0)\Delta x + \sin(\pi/8)\Delta x + \sin(\pi/4)\Delta x + \sin(3\pi/8)\Delta x \\ &= (\pi/8)(\sin(0) + \sin(\pi/8) + \sin(\pi/4) + \sin(3\pi/8)) \approx 0.790766. \end{aligned}$$

Here is the graph of the function and the approximate squares. We see that we overestimated the area under the curve.

Problem 14 (except c))

a) We have $t_1 = 0$, $t_2 = 10$, $t_3 = 20$, $t_4 = 30$, $t_5 = 40$, and $t_6 = 50$ as our sample points and $\Delta x = 10$. So, the distance is estimated by

$$10(182.9) + 10(168.0) + 10(106.6) + 10(99.8) + 10(124.5) + 10(176.1) \approx 8579.0 \text{ miles}.$$



b) We have this time $t_1 = 10$, $t_2 = 20$, $t_3 = 30$, $t_4 = 40$, $t_5 = 50$, and $t_6 = 60$ as our sample points and $\Delta x = 10$. So, the distance is estimated by

$$10(168.0) + 10(106.6) + 10(99.8) + 10(124.5) + 10(176.1) + 10(175.6) \approx 8506.0 \text{ miles.}$$