Problem 2

With $n = 6$, we obtain $\delta x = \pi/8$, $a = 0$, and $b = 3\pi/4$. Using the left endpoints, our sample points are

$$\begin{aligned} x_1 &= 0, & x_4 &= 3\pi/8, \\ x_2 &= \pi/8, & x_5 &= \pi/2, \\ x_3 &= \pi/4, & x_6 &= 5\pi/8. \end{aligned}$$

So, the Riemann sum is $\sum_{i=1}^{6} f(x_{i-1})\Delta x$ and we obtain the following estimate for the integral:

$$\int_0^{3\pi/4} \cos x \, dx \approx 1.033185.$$

The Riemann sum that we just computed represents an approximation of the integral of the function $f(x) = \cos x$ from $a = 0$ to $b = 3\pi/4$. It also represents the net area under the curve of $\cos x$.

Problem 6(c)

We have $a = -2$ and $b = 4$. We want $n = 6$ subintervals, so $\Delta x = 1$. The midpoints of each subintevals will be our sample points and they are

$$\begin{aligned} \overline{x}_1 &= -1.5, & \overline{x}_4 &= 1.5 \\ \overline{x}_2 &= -0.5, & \overline{x}_5 &= 2.5 \\ \overline{x}_3 &= 0.5 & \overline{x}_6 &= 3.5. \end{aligned}$$

So the integral of the function is approximated by

$$\begin{aligned} \int_{-2}^{4} f(x)\, dx &\approx \Delta x \Big(f(\overline{x}_1) + f(\overline{x}_2) + f(\overline{x}_3) + f(\overline{x}_4) + f(\overline{x}_5) + f(\overline{x}_6) \Big) \\ &= -1 - 1 + 1 + 1 + 0 - 0.5 = -0.5. \end{aligned}$$

Problem 18

The function is $f(x) = x\sqrt{1 + x^3}$ and we have $a = 2$, $b = 5$. So the limit represents

$$\int_2^5 x\sqrt{1 + x^3}\, dx.$$

Problem 22

Let n be the number of subintervals. We have $a = 1$ and $b = 4$, so $\Delta x = 3/n$. We also have that the right endpoints of each subinterval are $x_i = 1 + i\Delta x = 1 + 3i/n$. So, using the right endpoints rule, we know that

$$\int_1^4 (x^2 - 4x + 2)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x.$$

We have

$$\begin{aligned}
\sum_{i=1}^{n} f(x_i)\Delta x &= \frac{3}{n}\sum_{i=1}^{n}(1 + 3i/n)^2 - 4 - 12i/n + 2 \\
&= \frac{3}{n}\left[\sum_{i=1}^{n} 1 + 6i/n + 9i^2/n^2 - 4 - 12i/n + 2\right] \\
&= \frac{3}{n}\left[\sum_{i=1}^{n} -1 - 6i/n + 9i^2/n^2\right] \\
&= \frac{3}{n}\left[\sum_{i=1}^{n} \frac{9i^2 - 6in - n^2}{n^2}\right] \\
&= \frac{3}{n^3}\left[9\sum_{i=1}^{n} i^2 - 6n\sum_{i=1}^{n} i - \sum_{i=1}^{n} n^2\right] \\
&= \frac{3}{n^3}\left[\frac{3n(n+1)(2n+1)}{2} - 3n^2(n+1) - 3n^3\right] \\
&= \frac{18n^3 + 27n^2 + 9n}{n^3} - \frac{9n^3 + 9n^2}{n^3} - 3.
\end{aligned}$$

Taking the limit as $n \to \infty$, we obtain

$$\int_1^4 (x^2 - 4x + 2)\, dx = 6.$$