Problem 2, (a) and (c)

(a) We have $g(0) = 0$, $g(1) = 1/2$, $g(2) = 0$, $g(3) = -1/2$, $g(4) = 0$, $g(5) = 1/2$, and $g(6) = 1$.

(c) By the FTC part I, we have $g'(x) = f(x)$. We see that $g'(x)$ doesn't exist when $x = 2$ and $x = 6$, and is zero at $x = 1$ and $x = 3$. Those are the critical points. We can use the closed interval method to find the maximum and minimum value.

- The maximum value is the $\max\{g(0), g(1), g(2), g(3), g(6)\} = 1$.
- The minimum value is the $\min\{g(0), g(1), g(2), g(3), g(6)\} = -1/2$.

Problem 8

By the Fundamental Theorem of Calculus, we immediately have

$$g'(x) = \cos(x^2).$$

Problem 10

Again, from the Fundamental Theorem of Calculus, we have

$$h'(u) = \frac{\sqrt{t}}{t+1}.$$

Problem 12

Using a property of the integral, we can rewrite $R(y)$ as

$$R(y) = -\int_2^y t^3 \sin(t)\, dt.$$

Therefore, using the FTC, we obtain

$$R'(y) = -y^3 \sin(y).$$

Problem 14

Write $H(x)$ for $\int_1^x \frac{z^2}{z^4+1}\, dz$. Then the function $h(x)$ can be rewritten as

$$h(x) = H(\sqrt{x}).$$

From the Chain Rule, we find that $h'(x) = H'(\sqrt{x})\frac{d}{dx}(\sqrt{x})$. Using the FTC, we know that

$$H'(x) = \frac{x^2}{x^4+1}$$

and therefore, we obtain

$$h'(x) = \frac{(\sqrt{x})^2}{(\sqrt{x})^4+1}\left(\frac{d}{dx}(\sqrt{x})\right) = \frac{1}{2\sqrt{x}}\frac{x}{x^2+1} = \frac{\sqrt{x}}{2(x^2+1)}.$$

Problem 18

We rewrite the expression of y as

$$y = -\int_1^{\sin x} \sqrt{1+t^2}\,dt.$$

Writing $Y(x) = \int_1^x \sqrt{1+t^2}\,dt$, we can rewrite y as

$$y(x) = Y(\sin(x)).$$

From the Chain Rule, we obtain

$$y'(x) = Y'(\sin(x))\frac{d}{dx}\big(\sin(x)\big).$$

Using the FTC, we see that

$$Y'(x) = \sqrt{1+x^2}$$

and replacing x by $\sin(x)$, we obtain

$$y'(x) = \left(\sqrt{1+\sin^2(t)}\right)\cos(x)$$

Problem 20

An antiderivative of x^{100} is $x^{101}/101$. Thus, by FTC part 2, we have

$$\int_{-1}^{1} x^{100}\,dx = \left.\frac{x^{101}}{101}\right|_{-1}^{1} = \frac{2}{101}.$$

Problem 22

Using linearity, we have

$$\int_0^1 (1-8v^3+16v^7)\,dv = \int_0^1 dv - 8\int_0^1 v^3\,dv + 16\int_0^1 v^7\,dv.$$

Using the part 2 of the FTC, we have

$$\int_0^1 (1 - 8v^3 + 16v^7)\,dv = v\Big|_0^1 - 8\,\frac{v^4}{4}\Big|_0^1 + 16\,\frac{v^8}{8}\Big|_0^1 = 1 - 2 + 2 = 1.$$

Problem 28

We simply the integrand:

$$(4 - t)\sqrt{t} = 4\sqrt{t} - t^{3/2}.$$

An antiderivative of this last function is

$$\frac{8}{3}t^{3/2} - \frac{2}{5}t^{5/2}.$$

Therefore, from the FTC, we have

$$\begin{aligned}
\int_0^4 (4 - t)\sqrt{t}\,dt &= \left(\tfrac{8}{3}t^{3/2} - \tfrac{2}{5}t^{5/2}\right)\Big|_0^4 \\
&= \left(\tfrac{8}{3}(4)^{3/2} - \tfrac{2}{5}(4)^{5/2}\right) - \left(\tfrac{8}{3}(0)^{3/2} - \tfrac{5}{2}(0)^{5/2}\right) \\
&= \frac{64}{3} - \frac{64}{5} \\
&= \frac{64}{15}(5 - 3) \\
&= \frac{128}{15}.
\end{aligned}$$

Problem 30

We rewrite the expression of the integrand as

$$(3u - 2)(u + 1) = 3u^2 + u - 2.$$

An antiderivative for this integrand is

$$u^3 + \frac{u^2}{2} - 2u.$$

Therefore, from the FTC, we get

$$\begin{aligned}
\int_{-1}^2 (3u - 2)(u + 1)\,du &= \left(u^3 + \tfrac{1}{2}u^2 - 2u\right)\Big|_{-1}^2 \\
&= \Big(8 + 2 - 4\Big) - \Big(-1 + \tfrac{1}{2} + 2\Big) \\
&= 6 - \tfrac{3}{2} \\
&= \tfrac{9}{2}.
\end{aligned}$$

Problem 34

We have $(s^4+1)/s^2 = s^2 + 1/s^2$. Thus,

$$\int_1^2 \frac{s^4+1}{s^2}\,ds = \int_1^2 s^2\,ds + \int_1^2 (1/s^2)\,ds = \frac{s^3}{3}\bigg|_1^2 + \frac{-1}{s}\bigg|_1^2 = \frac{8-1}{3} + 1/2 = 11/6.$$

Problem 35

The expression of the integrand can be rewritten as

$$\frac{v^5+3v^6}{v^4} = v + 3v^2.$$

An antiderivative for this integrand is

$$\frac{v^2}{2} + v^3.$$

Therefore, form the FTC, we have

$$\begin{aligned}\int_1^2 \frac{v^5+3v^6}{v^4}\,dv &= \left(\tfrac{1}{2}v^2 + v^3\right)\Big|_1^2 \\ &= \left(2+8\right) - \left(\tfrac{1}{2}+1\right) \\ &= 10 - \tfrac{3}{2} \\ &= \tfrac{17}{2}.\end{aligned}$$

Problem 38

We divide the integral in two pars:

$$\int_{-2}^2 f(x)\,dx = \int_{-2}^0 f(x)\,dx + \int_0^2 f(x)\,dx.$$

According to the definition of the function $f(x)$, we have

$$\int_{-2}^2 f(x)\,dx = \int_{-2}^0 2\,dx + \int_0^2 (4-x^2)\,dx = 4 + 8 - 8/3.$$

So the final answer is $28/3$.

Problem 54

We rewrite $g(x)$ as followed:

$$g(x) = \int_{1-2x}^0 t\sin t\,dt + \int_0^{1+2x} t\sin t\,dt = -\int_0^{1-2x} t\sin t\,dt + \int_0^{1+2x} t\sin t\,dt.$$

Write

$$G(x) = \int_0^x t \sin t \, dt$$

so that

$$g(x) = -G(1-2x) + G(1+2x).$$

Using the Chain Rule, we get

$$g'(x) = -G'(1-2x)\frac{d}{dx}(1-2x) + G'(1+2x)\frac{d}{dx}(1+2x).$$

From the FTC, we have

$$G'(x) = x \sin x$$

so that

$$g'(x) = 2(1-2x)\sin(1-2x) + 2(1+2x)\sin(1+2x).$$

We can simply this expression using some trig. identities. In $g(x)$, we have the expression

$$2\sin(1-2x) + 2\sin(1+2x) = 4\sin(1)\cos(2x)$$

and the expression

$$-4x\sin(1-2x) + 4x\sin(1+2x) = 8x\sin(2x)\cos(1).$$

We thus obtain

$$g'(x) = 4\sin(1)\cos(2x) + 8x\cos(1)\sin(2x).$$

Problem 60

By the FTC (part 1), we have $F'(x) = f(t)$. So, the function is concave downward when $F'(x)$ varies from being decreasing (corresponding to the second derivative being negative). From the graph of f, we see that f is decreasing on the interval $(-1, 1)$. Thus, F is concave down on $(-1, 1)$.

Problem 75

By the FTC (part 1), we have

$$\frac{f(x)}{x^2} = \frac{1}{\sqrt{x}}.$$

Thus, isolating $f(x)$, we obtain $f(x) = x^{3/2}$. Now, using the FTC (part 2), we have

$$6 + \int_a^x t^{-1/2} \, dt = 2\sqrt{x} \quad \Rightarrow \quad 6 + 2\sqrt{x} - 2\sqrt{a} = 2\sqrt{x}.$$

We then find $2\sqrt{a} = 6$ and so $a = 9$.

The desire function and number a are $f(x) = x^{3/2}$ and $a = 9$.