Problem 5

We have

$$\int (x^{1.3} + 7x^{2.5})\, dx = \int x^{1.3}\, dx + 7\int x^{2.5}\, dx = \frac{x^{1.3+1}}{1.3+1} + 7\frac{x^{2.5+1}}{2.5+1} + C = \frac{x^{2.3}}{2.3} + 7\frac{x^{3.5}}{3.5} + C.$$

Problem 6

We can write $\sqrt[4]{x^5} = x^{5/4}$ and therefore

$$\int \sqrt[4]{x^5}\, dx = \frac{x^{5/4}}{5/4} + C = \frac{4}{5}x^{5/4} + C.$$

Problem 10

We have

$$\sqrt{t}(t^2 + 3t + 2) = t^{5/2} + 3t^{3/2} + 2t^{1/2}$$

and therefore

$$\int \sqrt{t}(t^2 + 3t + 2)\, dt = \int t^{5/2}\, dt + 3\int t^{3/2}\, dt + 2\int t^{1/2}\, dt = \frac{2}{7}t^{7/2} + \frac{6}{5}t^{5/2} + \frac{4}{3}t^{3/2} + C.$$

Problem 11

We have

$$\frac{1 + \sqrt{x} + x}{\sqrt{x}} = \frac{1}{\sqrt{x}} + 1 + \sqrt{x} = x^{-1/2} + 1 + x^{1/2}.$$

Therefore,

$$\int \frac{1 + \sqrt{x} + x}{\sqrt{x}}\, dx = \int x^{-1/2}\, dx + \int dx + \int x^{1/2}\, dx = 2x^{1/2} + x + \frac{2}{3}x^{3/2} + C.$$

Problem 14

We have

$$\sec t(\sec t + \tan t) = \sec^2 t + \sec t \tan t.$$

Therefore,

$$\int \sec t(\sec t + \tan t)\, dt = \int \sec^2 t\, dt + \int \sec t \tan t\, dt = \tan t + \sec t + C.$$

Problem 15

We have

$$\frac{1 - \sin^3 t}{\sin^2 t} = \frac{1}{\sin^2 t} - \sin t = \csc^2 t - \sin t.$$

Therefore,

$$\int \frac{1 - \sin^3 t}{\sin^2 t}\, dt = \int \csc^2 t\, dt - \int \sin t\, dt = -\cot t + \cos t + C.$$

Problem 18

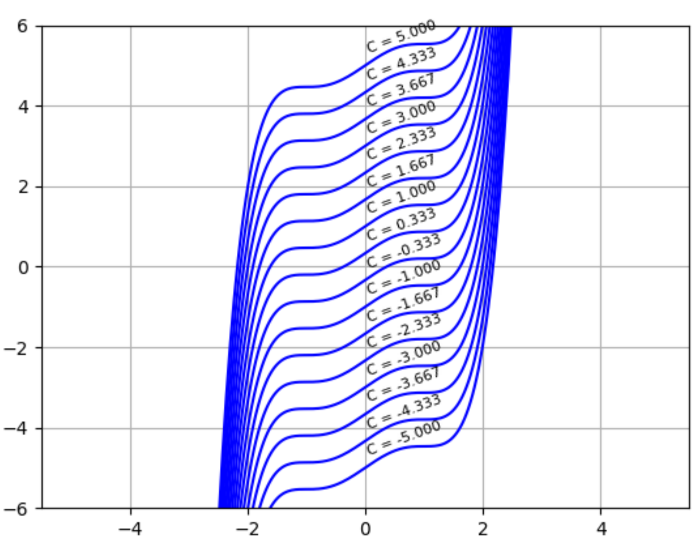
We have

$$(1 - x^2)^2 = 1 - 2x^2 + x^4$$

and so

$$\int (1 - x^2)^2\, dx = \int\, dx - 2\int x^2\, dx + \int x^4\, dx = x - \frac{2}{3}x^3 + \frac{x^5}{5} + C.$$

Here is the graph of several antiderivatives with different constants C.



Problem 41

When $x \leq 0$, we have

$$x - 2|x| = x + 2x = 3x$$

and when $x \geq 0$, we have

$$x - 2|x| = x - 2x = -x.$$

Therefore, the integral is

$$\begin{aligned}\int_{-1}^{2} (x - 2|x|)\,dx = \int_{-1}^{0} 3x\,dx - \int_{0}^{3} x\,dx &= 3\Big(\frac{0^2 - (-1)^2}{2}\Big) - \Big(\frac{3^2 - 0^2}{2}\Big) \\ &= -\frac{3}{2} - \frac{9}{2} = -6.\end{aligned}$$

Problem 58

(a) The velocity is given by

$$v(t) = \int 2t + 3\,dt = t^2 + 3t + C.$$

Now, $v(0) = -4$, so $C = -4$. We then get

$$v(t) = t^2 + 3t - 4 = (t+4)(t-1).$$

(b) The distance travelled during the interval is given by

$$\int_0^3 |v(t)|\,dt.$$

The function $v(t) = (t+4)(t-1)$ and therefore

$$|v(t)| = \begin{cases} -(t^2 + 3t - 4) & \text{if } 0 \leq t \leq 1 \\ t^2 + 3t - 4 & \text{if } 1 < t \leq 3. \end{cases}$$

We then obtain

$$\begin{aligned}\int_0^3 |v(t)|\,dt &= \int_0^1 -t^2 - 3t + 4\,dt + \int_1^3 t^2 + 3t - 4\,dt \\ &= \Big(-\frac{t^3}{3} - \frac{3}{2}t^2 + 4t\Big)\Big|_0^1 + \Big(\frac{t^3}{3} + \frac{3}{2}t^2 - 4t\Big)\Big|_1^3 \\ &= \frac{89}{6}\end{aligned}$$

Therefore, the total distance traveled is $\frac{89}{6} \approx 14.8333$ meters.