Problem 2

Let $u = 2x^2 + 3$, so that

$$\frac{du}{dx} = 4x \quad \Rightarrow \quad \frac{1}{4}du = xdx.$$

The integral becomes

$$\int x(2x^2+3)^4\,dx = \int u^4 \frac{du}{4} = \frac{u^5}{20} + C$$

and the answer is

$$\frac{(2x^2+3)^5}{20} + C.$$

Problem 4

Let $u = \sin\theta$. Then

$$\frac{du}{dx} = \cos\theta \quad \Rightarrow \quad du = \cos\theta\,dx.$$

There integral becomes

$$\int \sin^2\theta\cos\theta\,d\theta = \int u^2\,du = \frac{u^3}{3} + C$$

and the answer is

$$\frac{\sin^3\theta}{3} + C.$$

Problem 8

Let $u = x^3$, so that

$$\frac{du}{dx} = 3x^2 \quad \Rightarrow \quad \frac{1}{3}du = x^2\,dx.$$

Therefore, the integral becomes

$$\int x^2\sin(x^3)\,dx = \int \sin(u)\,\frac{du}{3} = -\tfrac{1}{3}\cos(u) + C.$$

The answer is then

$$-\tfrac{1}{3}\cos(x^3) + C.$$

Problem 10

Set $u = 1 + \cos t$, then

$$\frac{du}{dt} = -\sin(t) \quad \Rightarrow \quad -du = \sin(t)\,dt.$$

The integral then becomes

$$\int \sin t\sqrt{1+\cos t}\,dt = -\int \sqrt{u}\,du = -\tfrac{2}{3}u^{3/2} + C$$

and the final answer is

$$-\tfrac{2}{3}(1+\cos t)^{3/2} + C.$$

Problem 16

Let $u = \sqrt{x}$, so that

$$\frac{du}{dx} = \frac{1}{2\sqrt{x}}\,dx \quad \rightarrow \quad 2\,du = \frac{1}{\sqrt{x}}\,dx.$$

The integral then becomes

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int \sin(u)\,2du = -2\cos(u) + C.$$

The answer is then

$$-2\cos(\sqrt{x}) + C.$$

Problem 20

Let $u = x + 2$, then $du = dx$ and

$$\int x\sqrt{x+2}\,dx = \int (u-2)\sqrt{u}\,du = \int u^{3/2} - 2u^{1/2}\,du = \tfrac{2}{5}u^{5/2} - \tfrac{4}{3}u^{3/2} + C.$$

Then, the answer is

$$\tfrac{2}{5}(x+2)^{5/2} - \tfrac{4}{3}(x+2)^{3/2} + C.$$

Problem 26

Let $u = \tan x$, then

$$\frac{du}{dx} = \sec^2 x\, dx.$$

The integral then becomes

$$\int \frac{\sec^2 x}{\tan^2 x}\, dx = \int u^{-2}\, du = -u^{-1} + C$$

and therefore the final answer is

$$-\frac{1}{\tan x} + C.$$

Problem 30

Let $u = x^2 + 1$. Therefore, we have

$$\frac{du}{dx} = 2x \quad \Rightarrow \quad \tfrac{1}{2} du = x\, dx.$$

The integral becomes

$$\int x^3 \sqrt{x^2+1}\, dx = \int x^2 \sqrt{u} x\, dx = \tfrac{1}{2} \int x^2 \sqrt{u}\, du.$$

Since $u = x^2 + 1$, we have $x^2 = u - 1$ and therefore

$$\int x^2 \sqrt{u}\, du = \int (u-1)\sqrt{u}\, du = \int u^{3/2} - u^{1/2}\, du = \frac{1}{2}\Big(\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\Big) + C$$

and the final answer is

$$\frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C.$$

Problem 36

Set $u = 3t - 1$ so that $\frac{1}{3} du = dx$ and

$$x = 0 \rightarrow u = -1 \quad \text{ and } \quad x = 1 \rightarrow u = 2.$$

After a u-sub, the integral becomes

$$\int_{-1}^{2} u^{50} \frac{du}{3} = \frac{1}{3}\left(\frac{u^{51}}{51}\right)\Big|_{-1}^{2} = \frac{2^{51}+1}{153}.$$

Problem 38

Let $u = x^2$, so that $\frac{1}{2}du = x\,dx$. The integral then becomes

$$\int_0^{\sqrt{\pi}} x\cos(x^2)\,dx = \frac{1}{2}\int_0^{\pi} \cos(u)\,du = \sin u|_0^{\pi} = 0.$$

Problem 46

The function $f(x) = x^4 \sin x$ is an odd function since

$$f(-x) = (-x)^4 \sin(-x) = -x^4 \sin(x) = f(x).$$

We divide the integral in two

$$\int_{-\pi/3}^{\pi/3} x^4 \sin x\,dx = \int_{-\pi/3}^{0} x^4 \sin x\,dx + \int_0^{\pi/3} x^4 \sin x\,dx.$$

The first integral in the RHS, let $u = -x$, so that $-du = dx$ and therefore

$$\int_{-\pi/3}^{0} x^4 \sin x\,dx = -\int_{\pi/3}^{0} (-u)^4 \sin(-u)\,du.$$

Using the property of the integral:

$$-\int_{\pi/3}^{0} (-u)^4 \sin(-u)\,du = \int_0^{\pi/3} (-u)^4 \sin(-u)\,du.$$

Since $f(-u) = -f(u)$, we get that

$$\int_0^{\pi/3} (-u)^4 \sin(-u)\,du = -\int_0^{\pi/3} u^4 \sin u\,du.$$

Recall that the value of the integral does not depend on the name of the variable, therefore, we have

$$\int_0^{\pi/3} u^4 \sin u\,du = \int_0^{\pi/3} x^4 \sin x\,dx$$

and so

$$\int_{-\pi/3}^{\pi/3} x^4 \sin(x)\,dx = -\int_0^{\pi/3} x^4 \sin x\,dx + \int_0^{\pi/3} x^4 \sin x\,dx = 0.$$