Problem 8

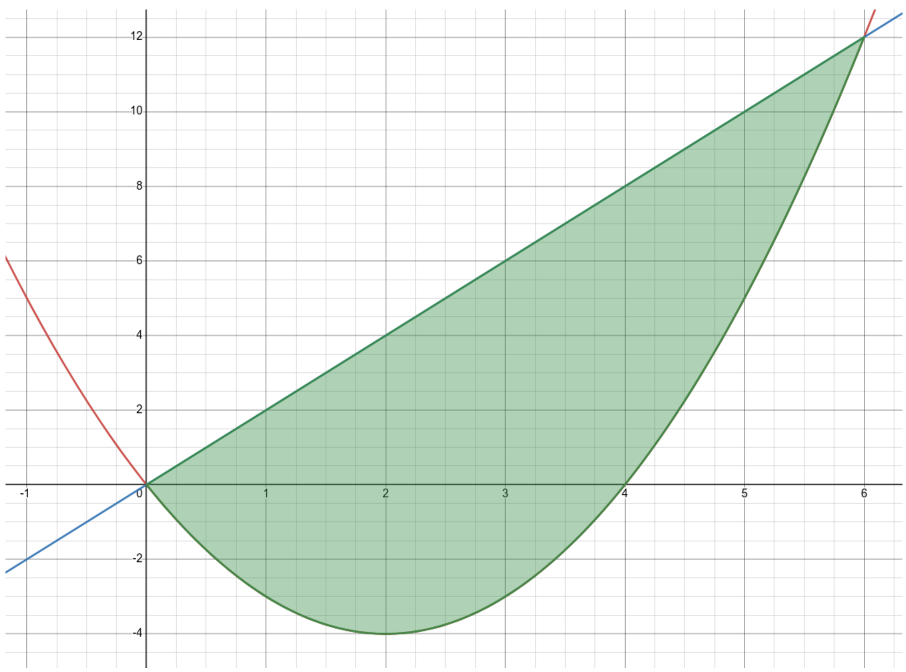
The intersections between $x^2 - 4x$ and $2x$ is given by the solutions to

$$x^2 - 4x = 2x \iff x^2 - 6x = 0 \iff x = 6 \text{ or } x = 0.$$

To have

$$x^2 - 4x \leq 6x \iff x(x-6) \leq 0$$

the value of x should be between 0 and 6 ($0 \leq x \leq 6$). Therefore, the region is enclosed by the curve $6x$ (top/ceiling) and $x^2 - 4x$ (bottom/floor) from $x = 0$ to $x = 6$.

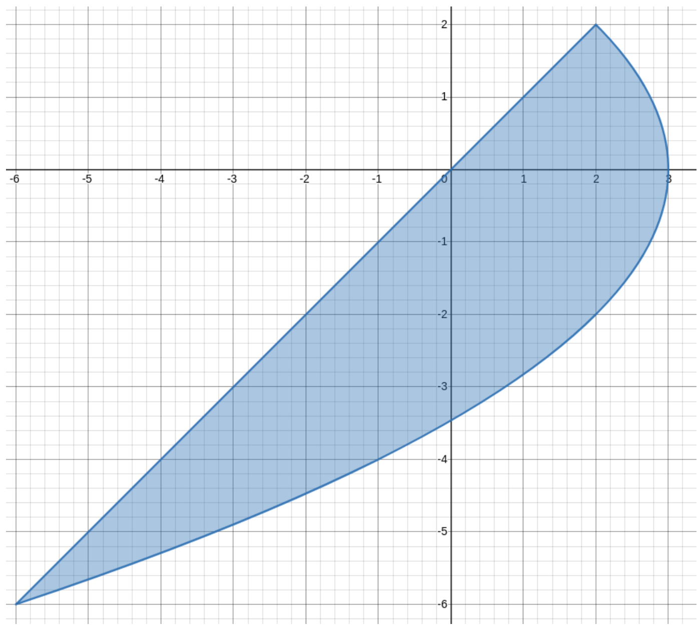


Therefore, the area of the region is given by

$$\int_0^6 2x - (x^2 - 4x)\,dx = \int_0^6 6x - x^2\,dx = \left(3x^2 - \frac{x^3}{3}\right)\Big|_0^6 = 36.$$

Problem 12

The sketch of the region is the following:



The intersections between the two curves are

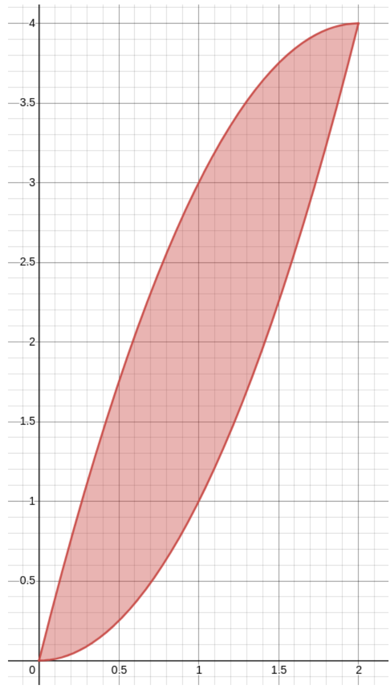
$$4y + y^2 = 12 \iff y^2 + 4y - 12 = 0 \iff y = -6 \text{ or } y = 2.$$

We now have

$$A(S) = \int_{-6}^{2} x_R - x_L \, dy = \int_{-6}^{2} 3 - y^2/4 - y \, dy = 64/3.$$

Problem 14

The sketch of the region:



The intersections between the two curves are

$$x^2 = 4x - x^2 \iff x^2 - 2x = 0 \iff x = 0 \text{ or } x = 2.$$

So, we have

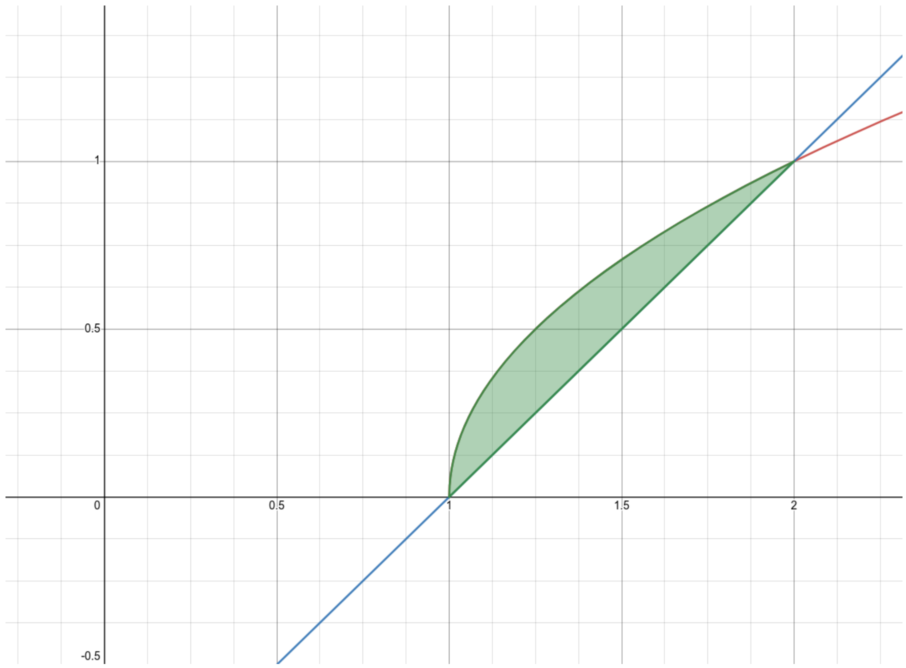
$$A(S) = \int_0^2 (4x - x^2) - (x^2)\, dx = 8/3.$$

Problem 18

The second curve is $y = x - 1$. The intersections between the two curves are

$$\sqrt{x-1} = x - 1 \iff x - 1 = (x-1)^2 \iff (x-2)(x-1) = 0$$

and therefore $x = 1$ or $x = 2$. Here is the region between the two curves.



We have $\sqrt{x-1} \geq x - 1$ for $1 \leq x \leq 2$. The area of the region is therefore

$$\int_1^2 \sqrt{x-1} - (x-1)\,dx = \int_1^2 \sqrt{x-1}\,dx - \int_1^2 x - 1\,dx$$

For the first integral, use a change of variable. Set $u = x - 1$, then $du = dx$ and

$$\int_1^2 \sqrt{x-1}\,dx = \int_0^1 \sqrt{u}\,du = \left(\frac{2}{3}u^{3/2}\right)\Big|_0^1 = \frac{2}{3}.$$

Also, we have

$$\int_1^2 (x-1)\,dx = \left(\frac{x^2}{2} - x\right)\Big|_1^2 = (2-2) - (1/2 - 1) = 1/2.$$

Therefore, the area is

$$\int_1^2 \sqrt{x-1} - (x-1)\,dx = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}.$$