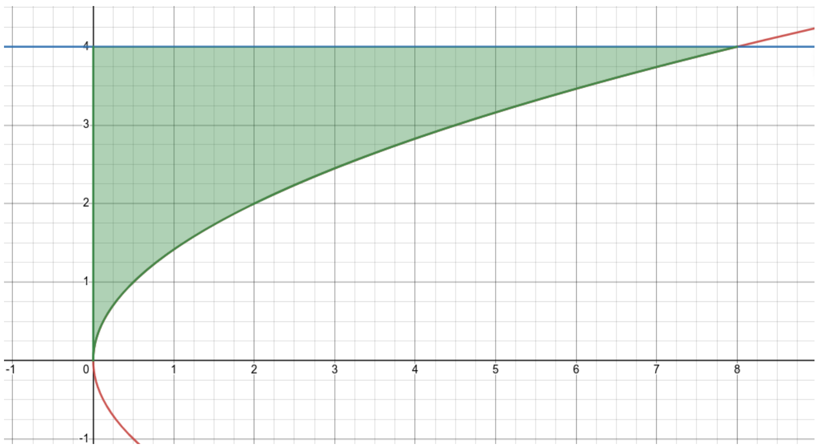
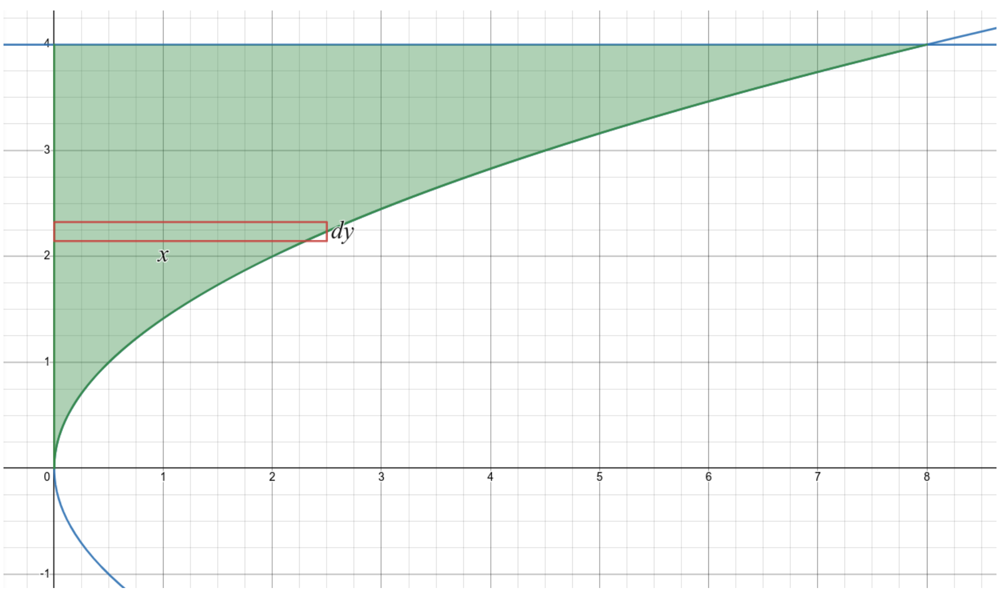
Problem 6

The curve is a parabola, $x = y^2/2$. The y values are bounded by $y = 0$ (because $x = 0$ implies that $y = 0$) and $y = 4$. Therefore, the region is given by $0 \leq x \leq y^2/2$ and $0 \leq y \leq 4$.



The rotation is about the y-axis. We therefore draw a small horizontal rectangle with height dy and width x.



After rotation, the radius of the disk created is x and the height is dy. Therefore, the volume is

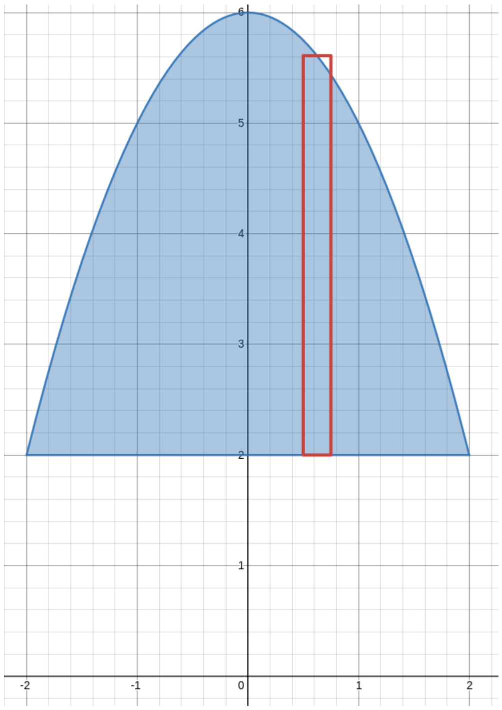
$$\int_0^4 \pi(\text{radius})^2 dy = \int_0^4 \pi x^2 \, dy.$$

But now, $x = y^2/2$, and therefore

$$\int_0^4 \pi x^2 \, dy = \int_0^4 \pi \frac{y^4}{4} \, dy = \pi \left(\frac{y^5}{20} \right) \Big|_0^4 = \frac{256\pi}{5}.$$

Problem 8

Here is the sketch of the region to rotate about the x-axis and a gif of the rotation[1]:



(a) Region to rotate

(b) Rotation of the region

We will use the washer method. The inner radius is $r_{in} = y = 2$ and the outer radius is $r_{out} = y = 6 - x^2$. The values of x start at $x = -2$ and ends at $x = 2$. So, by the washer method, we obtain

$$V(S) = \int_{-2}^{2} \pi r_{out}^2 - \pi r_{in}^2 \, dx = \pi \int_{-2}^{2} ((6 - x^2)^2 - 2^2) \, dx = 384/5.$$

[1]The gif will work if you open the pdf with Adobe Acrobat Reader.