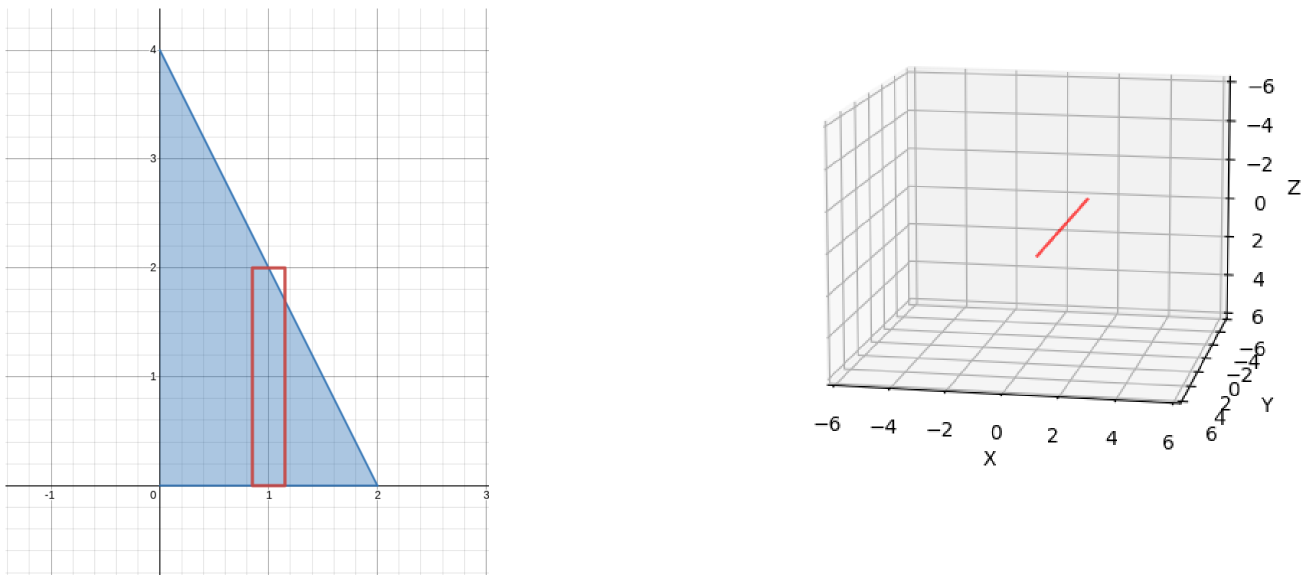
Problem 16

Here is the sketch of the region to rotate about the axis $x = 3$:



(a) Region to rotate (b) Rotation of the region

We will use the cylindrical shells method. The radius is $x + 1$ and the height is y and the limits are $0 \leq x \leq 2$. Thus, we get

$$V(S) = \int_0^2 2\pi(x+1)y\,dx = 2\pi\int_0^2 (x+1)(4-2x)\,dx = 20/3.$$

Problem 18

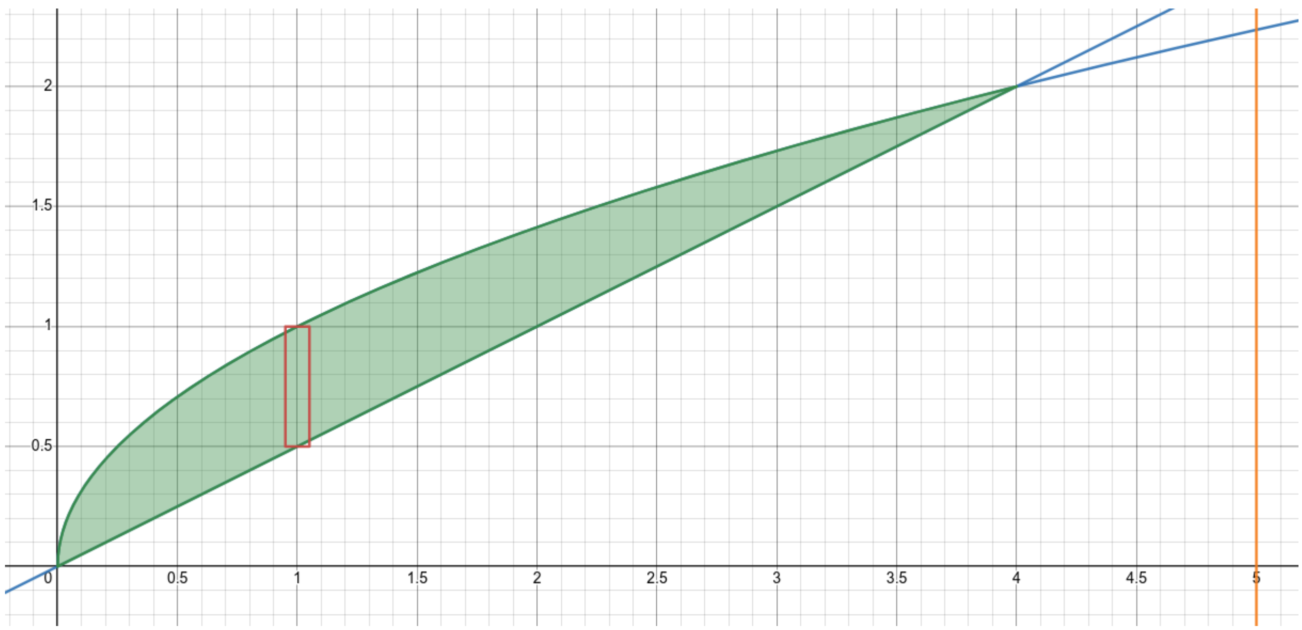
The region is bounded by the curves

$$y = \sqrt{x} \quad \text{and} \quad x = 2y.$$

Therefore, the curve meets when

$$\sqrt{x} = \frac{x}{2} \iff x = \frac{x^2}{4} \iff \frac{1}{4}x(x-4) = 0 \iff x = 0 \text{ or } x = 4.$$

A sketch of the region is presented below with a typical rectangle to generate the spherical shell:



After rotating about the line $x = 5$, we obtain a cylindrical shell with

- height: $\sqrt{x} - \frac{x}{2}$;
- radius: $5 - x$;
- thickness: dx.

Therefore, the volume is given by

$$\int_a^b 2\pi(\text{radius})(\text{height})\,dx = \int_0^4 2\pi(5-x)\Big(\sqrt{x} - \frac{x}{2}\Big)\,dx$$

The value of this integral is the volume of the solid of revolution. Therefore, the volume of the solid of revolution is $\frac{136}{15}\pi$.

Problem 30

The radius is y and the height is $x = \sqrt{y-1}$ and the limits are $1 \leq y \leq 5$. So, the solid is obtained by rotating the following region around the x axis: