Created by Pierre-O. Parisé
2023/22/02, Spring 2023

Last name: ______________________

First name: ______________________

Section: ______________________

Instructions:

- Make sure to write your complete name on your copy.
- You must answer all eight (8) questions below and write your answers directly on the questionnaire.
- You have 75 minutes to complete the exam.
- When you are done (or at the end of the 75min period), return your copy.
- Devices such as smartphones, cellphones, laptops, tablets, e-readers, ipods, gameboys (and, you know, any other electronic devices that I haven't thought of) may not be used during the exam.
- You can not use a calculator.
- **Turn off your cellphones during the exam**.
- Lecture notes and the textbook are not allowed during the exam.
- You must show ALL your work to have full credit. An answer without justification is worth no points (except if it is mentioned explicitly in the question not to justify).
- Draw a square around your final answer.

Your Signature: ______________________

May the Force be with you! Pierre Parisé

University of Hawai'i

Question 1 (8 pts)

The table shows the distance travelled by a bicyclist on a straight line after accelerating from rest.

Time in seconds	Total distance in feet
0	0
1	2
2	4
3	8
4	15
5	30
6	52
7	76
8	101

(a) (2 points) Calculate the average speed between 2 and 6 seconds.

$$\frac{\Delta s}{\Delta t} = \frac{52-4}{6-2} = \frac{48}{4} = \boxed{12 \text{ ft/s}}$$

(b) (3 points) Compare the average speed of the interval between 0 second and 1 second, and the interval between 1 second and 2 seconds. Between these two intervals, which one has the highest average speed?

$$[0,1]: \quad \frac{\Delta s}{\Delta t} = \frac{2-0}{1-0} = 2 \text{ ft/s}$$

$$[1,2]: \quad \frac{\Delta s}{\Delta t} = \frac{4-2}{2-1} = 2 \text{ ft/s}$$

$\rightarrow$ same.

(c) (3 points) Estimate the average acceleration of the bicyclist at 7 seconds. (Hint: The average acceleration can be calculated using two average speeds.)

1st $[6,7]$: $$\frac{\Delta s}{\Delta t} = \frac{76-52}{7-6} = 24 \text{ ft/s}$$

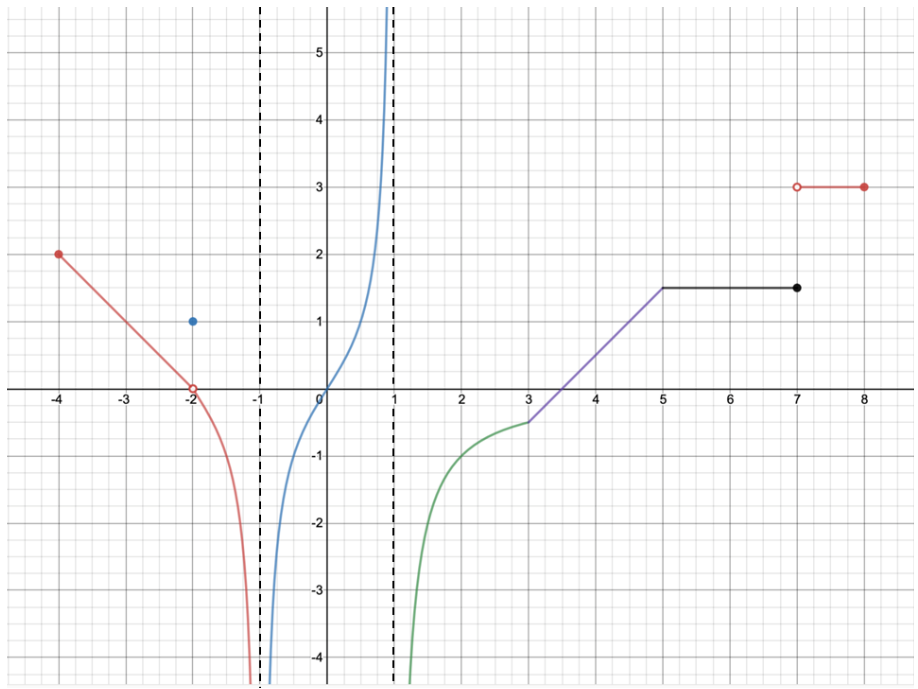
2nd $[7,8]$: $$\frac{\Delta s}{\Delta t} = \frac{101-76}{8-7} = 25 \text{ ft/s}$$

So,

$$\text{av. acc} = \frac{25-24}{8-6} = \boxed{\frac{1}{2} \text{ ft/s}^2}$$

Question 2 (15 pts)

The graph of a function f is given below. Assume f has vertical asymptotes at $x = -1$ and $x = 1$. **No justification needed for this problem.**



(a) (6 points) Evaluate each of the following limits, or say the limit does not exist. If the limit is either ∞ or $-\infty$, specify which (rather than just saying 'does not exist').

1. $\lim_{x \to -2} f(x)$ = 0
2. $\lim_{x \to -1^-} f(x)$ = $-\infty$
3. $\lim_{x \to 1} f(x)$ DNE.
4. $\lim_{x \to 7^-} f(x)$ = 1.5
5. $\lim_{x \to 7^+} f(x)$ = 3
6. $\lim_{x \to 7} f(x)$ DNE

(b) (3 points) For which (if any) values in the interval $[-4, 8]$ is the function f not continuous?

−2, −1, 1, 7.

(c) (3 points) For which (if any) values in the interval $[-4, 8]$ is f differentiable but not continuous?

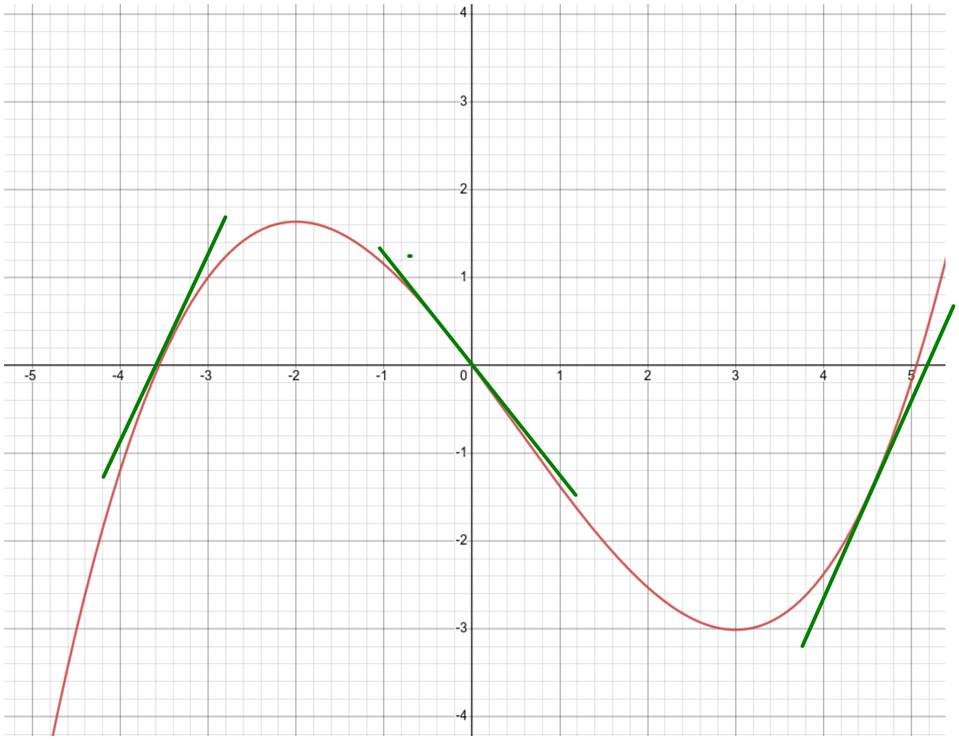
NONE.

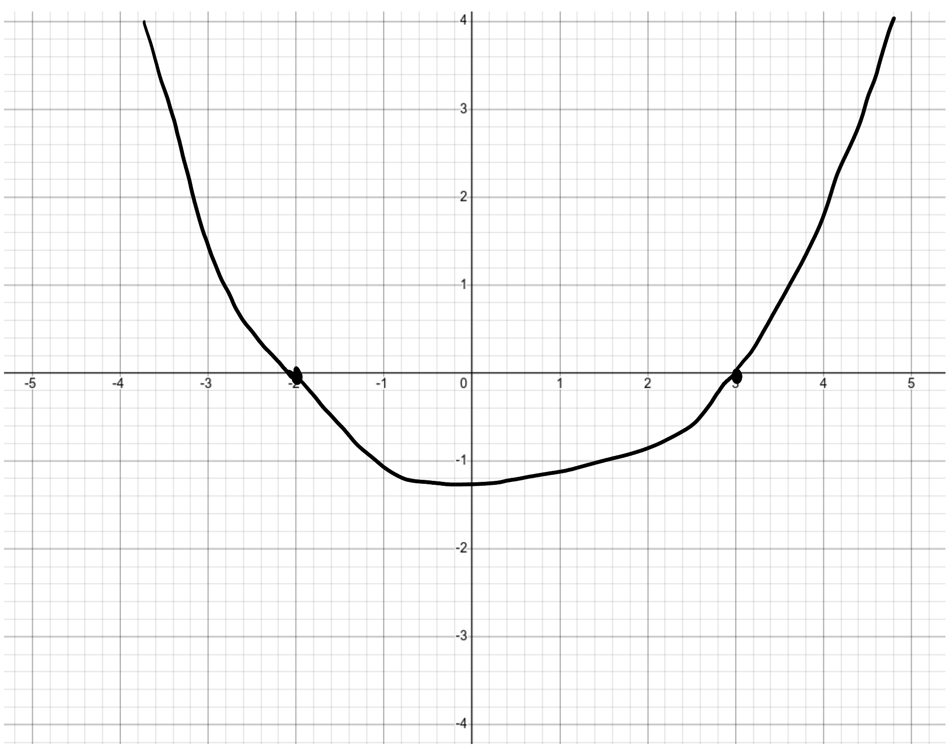
(d) (3 points)For which (if any) values in the interval $[-4, 8]$ is f continuous but not differentiable?

3, 5.

Question 3 (5 pts)

The graph of a function is given below. **Roughly** sketch the graph of the derivative in the blank axes.





Question 4 **(20 pts)**

Evaluate the following limits. You may not use L'Hospital's rule, i.e., if you use L'Hospital's rule, you will not get points.

(a) (5 points) $\lim_{x \to 1} (x^2 + x)(x + 1)$.

$$= \left(\lim_{x \to 1} x^2 + x\right)\left(\lim_{x \to 1} x + 1\right)$$

$$= (1^2 + 1)(1 + 1)$$

$$= \boxed{4}$$

(b) (5 points) $\lim_{x \to 0} \dfrac{x^2 - 3x - 4}{x + 1}$.

Subst.

$$= \frac{0^2 - 3(0) - 4}{0 + 1}$$

$$= \frac{-4}{1} = \boxed{-4}$$

(c) (5 points) $\lim_{x\to 0} \frac{\sqrt{3x^2+16}-4}{x^2}$.

$$= \lim_{x\to 0} \frac{\left(\sqrt{3x^2+16}-4\right)\left(\sqrt{3x^2+16}+4\right)}{x^2\left(\sqrt{3x^2+16}+4\right)}$$

$$= \lim_{x\to 0} \frac{3x^2+16-16}{x^2\left(\sqrt{3x^2+16}+4\right)}$$

$$= \lim_{x\to 0} \frac{3\cancel{x^2}}{\cancel{x^2}\left(\sqrt{3x^2+16}+4\right)} = \lim_{x\to 0} \frac{3}{\left(\sqrt{3x^2+16}+4\right)}$$

$$\overset{\text{subs.}}{=} \boxed{\frac{3}{8}}$$

(d) (5 points) $\lim_{x\to 0} \frac{\cos x \sin x}{x}$.

$$= \left(\lim_{x\to 0} \cos x\right)\left(\lim_{x\to 0} \frac{\sin x}{x}\right)$$

$$= \cos(0) \cdot 1$$

$$= 1 \cdot 1$$

$$= \boxed{1}$$

Question 5 **(15 pts)**

(a) (10 points) Using *the definition of derivative* (also called the limit process), find the derivative of the function $f(x) = \dfrac{1}{x+4}$.

You will NOT get any credit unless you use the definition of the derivative!

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{x+h+4} - \frac{1}{x+4}}{h}$$

$$= \lim_{h \to 0} \frac{x+4 - (x+h+4)}{(x+h+4)(x+4)h}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{(x+h+4)(x+4)\cancel{h}}$$

$$= \lim_{h \to 0} \frac{-1}{(x+h+4)(x+4)} = \boxed{\frac{-1}{(x+4)^2}}$$

(b) (5 points) Using the function in (a), find the equation of the tangent line to $y = f(x)$ at $(0, \frac{1}{4})$.

$$y - \frac{1}{4} = f'(0)(x-0)$$

$$\Rightarrow y - \frac{1}{4} = \frac{-1}{(0+4)^2}(x)$$

$$\Rightarrow \boxed{y = \frac{-1}{16}x + \frac{1}{4}}$$

Question 6 **(12 pts)**

Let $f(x)$ be defined by

$$f(x) = \begin{cases} (x-A)^2+2 & \text{if } x<2 \\ 3 & \text{if } x=2 \\ A+x & \text{if } x>2 \end{cases}$$

(a) (8 points) Find all values of A so that $\lim_{x\to 2} f(x)$ exists.

The limit exists if

$$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x)$$

$$\Rightarrow \lim_{x\to 2^-} \left((x-A)^2+2\right) = \lim_{x\to 2^+} A+x$$

$$\Rightarrow (2-A)^2 \cancel{+2} = A \cancel{+2}$$

$$\Rightarrow 4-4A+A^2-A=0$$

$$\Rightarrow A^2-5A+4=0$$

$$\Rightarrow (A-4)(A-1)=0 \Rightarrow \boxed{A=4} \text{ or } \boxed{A=1}$$

(b) (4 points) Find all possible values of A so that $f(x)$ is continuous at $x=2$, or show that none exist. Justify your answer.

To be cont., the limit must exist (at least).

$\underline{A=4}$ $\quad \lim_{x\to 2} f(x) = 4+2 = 6 \neq 3 = f(2)$

not cont.

$\boxed{A=1}$ $\quad \lim_{x\to 2} f(x) = 1+2 = 3 = f(2)$

cont ✓

Question 7 (15 pts)

Differentiate the following functions. You are not required to simplify your answers.

(a) (5 points) $g(x) = x^3 + x\sec x + \cos x$.

$$g'(x) = 3x^2 + (x)'\sec x + x(\sec x)' - \sin x$$

$$= 3x^2 + \sec x + x\sec x\tan x - \sin x$$

(b) (5 points) $f(x) = \dfrac{x^2+x}{\sqrt{x}}$. $= \dfrac{x^2+x}{x^{1/2}} = x^{3/2} + x^{1/2}$

$$f'(x) = \frac{3}{2}x^{1/2} + \frac{1}{2}x^{-1/2}$$

(c) (5 points) $h(x) = \sqrt{4\sin(\pi x) + 3\tan(x^2)}$.

$$h'(x) = \frac{1}{2}\left(4\sin(\pi x) + 3\tan x^2\right)^{-1/2} \cdot \left(4\sin\pi x + 3\tan x^2\right)'$$

$$= \text{"} \qquad \text{"} \cdot \left(4\cos(\pi x)\,\pi + 3\sec^2(x^2)\,2x\right)$$

$$= \frac{1}{2}\left(4\sin(\pi x) + 3\tan x\right)^{-1/2}\left(4\pi\cos(\pi x) + 6x\sec^2(x^2)\right)$$

QUESTION 8 **(10 pts)**

You are given the following implicit equation describing a circle: $x^2 + 2x + y^2 = 4$.

(a) (8 points) **Use implicit differentiation** to find an equation of the tangent line to the circle passing through the point $(1, 1)$. A solution without using implicit differentiation will not be credited.

$\boxed{y = f(x)}$

$$\frac{d}{dx}(x^2 + 2x + y^2) = \frac{d}{dx}(4)$$

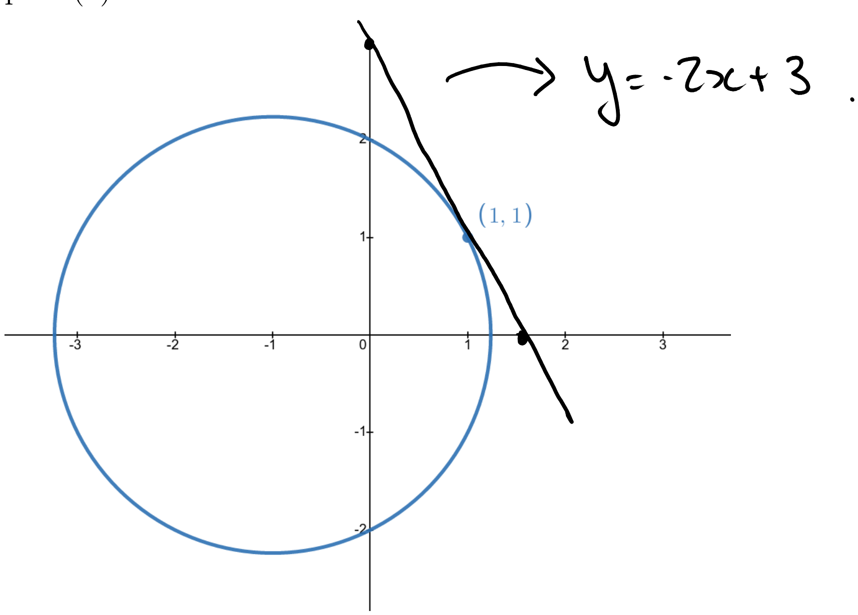
$$\Rightarrow 2x + 2 + 2yy' = 0$$

$$\Rightarrow 2 + 2 + 2y' = 0$$

$$\Rightarrow y' = -2$$

Pt. slope: $y - 1 = -2(x - 1) \Rightarrow \boxed{y = -2x + 3}$

(b) (2 points) The circle is drawn below. Sketch the graph of the tangent line obtained in part (a).



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For officials use only:

Question:	1	2	3	4	5	6	7	8	Total
Points:	8	15	5	20	15	12	15	10	100
Score:									