

Last name: _____

First name: _____

Section: _____

Instructions:

- Make sure to write your complete name on your copy.
- You must answer all eight (8) questions below and write your answers directly on the questionnaire.
- You have 75 minutes to complete the exam.
- When you are done (or at the end of the 75min period), return your copy.
- Devices such as smartphones, cellphones, laptops, tablets, e-readers, ipods, gameboys (and, you know, any other electronic devices that I haven't thought of) may not be used during the exam.
- You can not use a calculator.
- **Turn off your cellphones during the exam.**
- Lecture notes and the textbook are not allowed during the exam.
- You must show ALL your work to have full credit. An answer without justification is worth no points (except if it is mentioned explicitly in the question not to justify).
- Draw a square around your final answer.

Your Signature: _____

MAY THE FORCE BE WITH YOU!

PIERRE PARISÉ

UNIVERSITY
OF HAWAI'I



QUESTION 1

(8 pts)

The table shows the distance travelled by a bicyclist on a straight line after accelerating from rest.



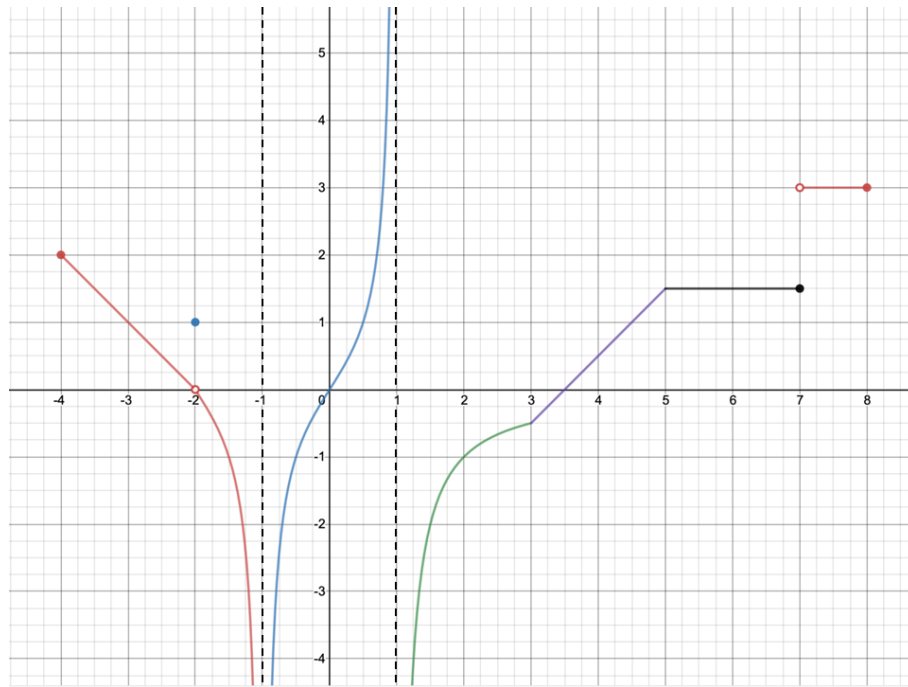
Time in seconds	Total distance in feet
0	0
1	2
2	4
3	8
4	15
5	30
6	52
7	76
8	101

- (a) (2 points) Calculate the average speed between 2 and 6 seconds.
- (b) (3 points) Compare the average speed of the interval between 0 second and 1 second, and the interval between 1 second and 2 seconds. Between these two intervals, which one has the highest average speed?
- (c) (3 points) Estimate the average acceleration of the bicyclist at 7 seconds.
(Hint: The average acceleration can be calculated using two average speeds.)

QUESTION 2

(15 pts)

The graph of a function f is given below. Assume f has vertical asymptotes at $x = -1$ and $x = 1$. No justification needed for this problem.



(a) (6 points) Evaluate each of the following limits, or say the limit does not exist. If the limit is either ∞ or $-\infty$, specify which (rather than just saying ‘does not exist’).

1. $\lim_{x \rightarrow -2} f(x)$

4. $\lim_{x \rightarrow 7^-} f(x)$

2. $\lim_{x \rightarrow -1^-} f(x)$

5. $\lim_{x \rightarrow 7^+} f(x)$

3. $\lim_{x \rightarrow 1} f(x)$

6. $\lim_{x \rightarrow 7} f(x)$

(b) (3 points) For which (if any) values in the interval $[-4, 8]$ is the function f not continuous?

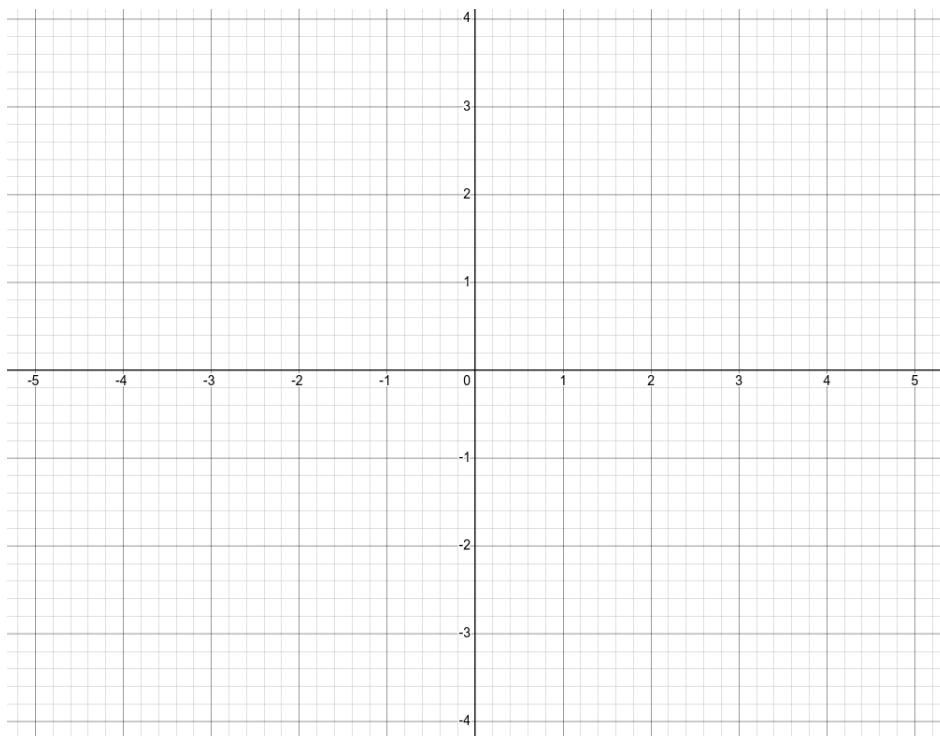
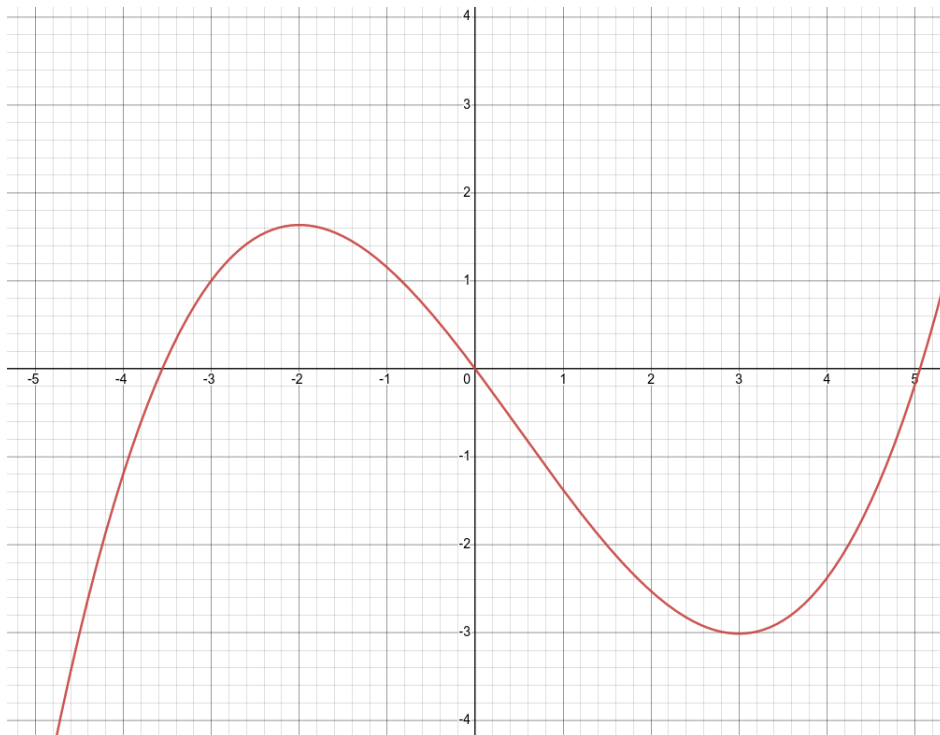
(c) (3 points) For which (if any) values in the interval $[-4, 8]$ is f differentiable but not continuous?

(d) (3 points) For which (if any) values in the interval $[-4, 8]$ is f continuous but not differentiable?

QUESTION 3

(5 pts)

The graph of a function is given below. **Roughly** sketch the graph of the derivative in the blank axes.



QUESTION 4

(20 pts)

Evaluate the following limits. You may not use L'Hospital's rule, i.e., if you use L'Hospital's rule, you will not get points.

(a) (5 points) $\lim_{x \rightarrow 1} (x^2 + x)(x + 1)$.

(b) (5 points) $\lim_{x \rightarrow 0} \frac{x^2 - 3x - 4}{x + 1}$.

(c) (5 points) $\lim_{x \rightarrow 0} \frac{\sqrt{3x^2 + 16} - 4}{x^2}$.

(d) (5 points) $\lim_{x \rightarrow 0} \frac{\cos x \sin x}{x}$.

QUESTION 5 (15 pts)

- (a) (10 points) Using *the definition of derivative* (also called the limit process), find the derivative of the function $f(x) = \frac{1}{x+4}$.

You will NOT get any credit unless you use the definition of the derivative!

- (b) (5 points) Using the function in (a), find the equation of the tangent line to $y = f(x)$ at $(0, \frac{1}{4})$.

QUESTION 6

(12 pts)

Let $f(x)$ be defined by

$$f(x) = \begin{cases} (x - A)^2 + 2 & \text{if } x < 2 \\ 3 & \text{if } x = 2 \\ A + x & \text{if } x > 2 \end{cases}$$

(a) (8 points) Find all values of A so that $\lim_{x \rightarrow 2} f(x)$ exists.

(b) (4 points) Find all possible values of A so that $f(x)$ is continuous at $x = 2$, or show that none exist. Justify your answer.

QUESTION 7

(15 pts)

Differentiate the following functions. You are not required to simplify your answers.

(a) (5 points) $g(x) = x^3 + x \sec x + \cos x$.

(b) (5 points) $f(x) = \frac{x^2 + x}{\sqrt{x}}$.

(c) (5 points) $h(x) = \sqrt{4 \sin(\pi x) + 3 \tan(x^2)}$.

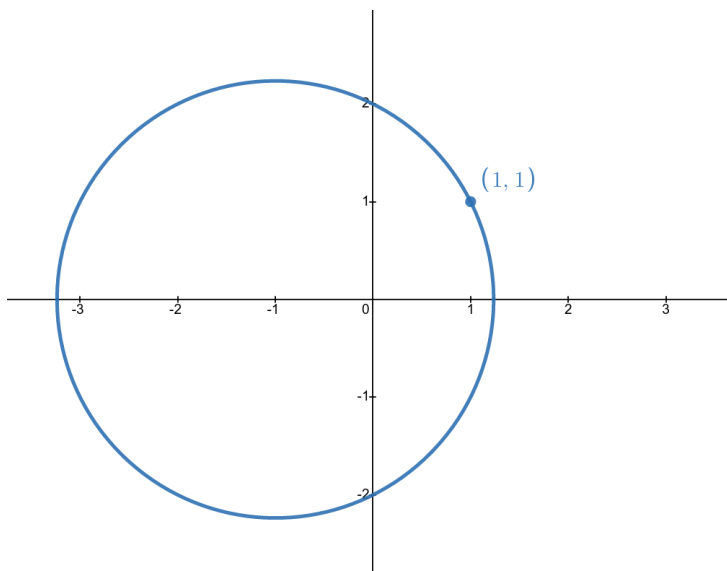
QUESTION 8

(10 pts)

You are given the following implicit equation describing a circle: $x^2 + 2x + y^2 = 4$.

- (a) (8 points) **Use implicit differentiation** to find an equation of the tangent line to the circle passing through the point $(1, 1)$. A solution without using implicit differentiation will not be credited.

- (b) (2 points) The circle is drawn below. Sketch the graph of the tangent line obtained in part (a).



DO NOT WRITE ON THIS PAGE.

For officials use only:

Question:	1	2	3	4	5	6	7	8	Total
Points:	8	15	5	20	15	12	15	10	100
Score:									