

Midterm 1

Name (Last, First): _____ Section: _____

Question:	1	2	3	4	5	6	7	8	Total
Points:	6	16	14	16	5	17	16	10	100
Score:									

Instructions:

- Write your full name (last, first) and section number above.
- Answer all the questions below and show your work.
- No electronic devices are to be used during the exam (this includes calculators).
- The exam is closed book and closed notes.
- **Do not use L'Hôpital's rule anywhere on this exam.**
- Turn in your exam at the end of the period.

Good luck!

Sign below to acknowledge that you have read and agree to the above instructions.

Signature: _____

Question 1

A bottle rocket was equipped with an altimeter and launched directly upward. Its altitude was recorded every second in the following table.

Time (seconds)	Altitude (meters)
0	0
1	4
2	9
3	15
4	22
5	30
6	38

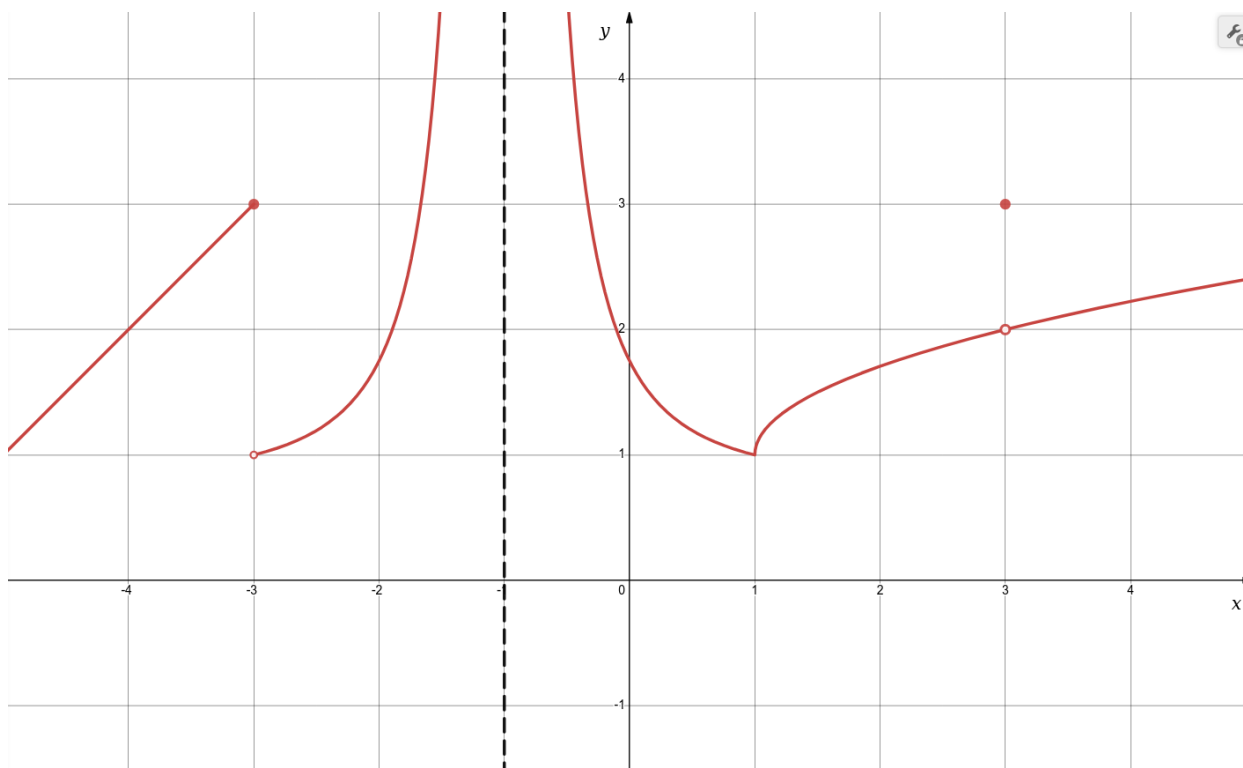
(a) (1 points) What was the bottle rocket's average velocity between 2 and 5 seconds?

(b) (2 points) Is the rocket moving faster, on average, between 2 and 5 seconds or between 5 and 6 seconds?

(c) (3 points) Estimate the bottle rocket's instantaneous velocity at 4 seconds. Make sure the method you are using is clear, and include units in your answer.

Question 2

The graph of a function f is shown below. Assume f has a vertical asymptote at $x = -1$.



(a) (6 points) Evaluate each of the following limits. If the limit is ∞ or $-\infty$, specify which. If the limit does not exist and is not ∞ or $-\infty$, then write DNE.

1. $\lim_{x \rightarrow 3} f(x)$

4. $\lim_{x \rightarrow -3^-} f(x)$

2. $\lim_{x \rightarrow -3} f(x)$

5. $\lim_{x \rightarrow -3^+} f(x)$

3. $\lim_{x \rightarrow -1^+} f(x)$

6. $\lim_{x \rightarrow -1} f(x)$

(b) (3 points) For which values (if any) in the interval $[-4, 4]$ is the function f not continuous?

(c) (3 points) For which values (if any) in the interval $[-4, 4]$ is the function f continuous, but not differentiable?

(d) (4 points) On which intervals is the function f decreasing? On which intervals is f increasing?

Question 3

Evaluate the following limits. If the limit does not exist, write DNE.

(a) (2 points) $\lim_{x \rightarrow 1} (2x^3 + x^2 - 3)$

(b) (4 points) $\lim_{x \rightarrow 1} \sqrt{x^2 + 8}$.

(c) (4 points) $\lim_{x \rightarrow 2} \frac{x^2 - 3x + 2}{x - 2}$.

(d) (4 points) $\lim_{x \rightarrow 0} \frac{x}{\sqrt{x+1} - 1}$.

Question 4

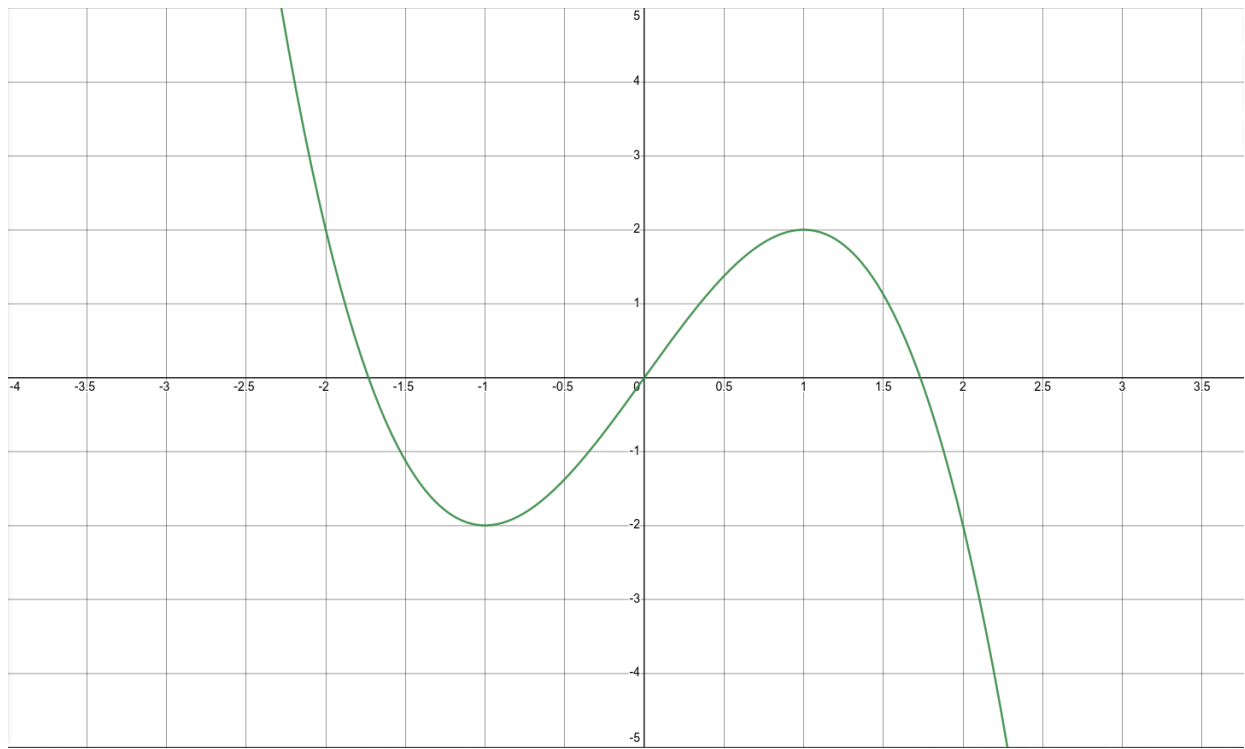
Let $f(x) = \frac{1}{x^2}$.

- (a) (12 points) Use the definition of the derivative (that is the limit process) to compute $f'(x)$. **You will not get any credit unless you use the definition of the derivative.**

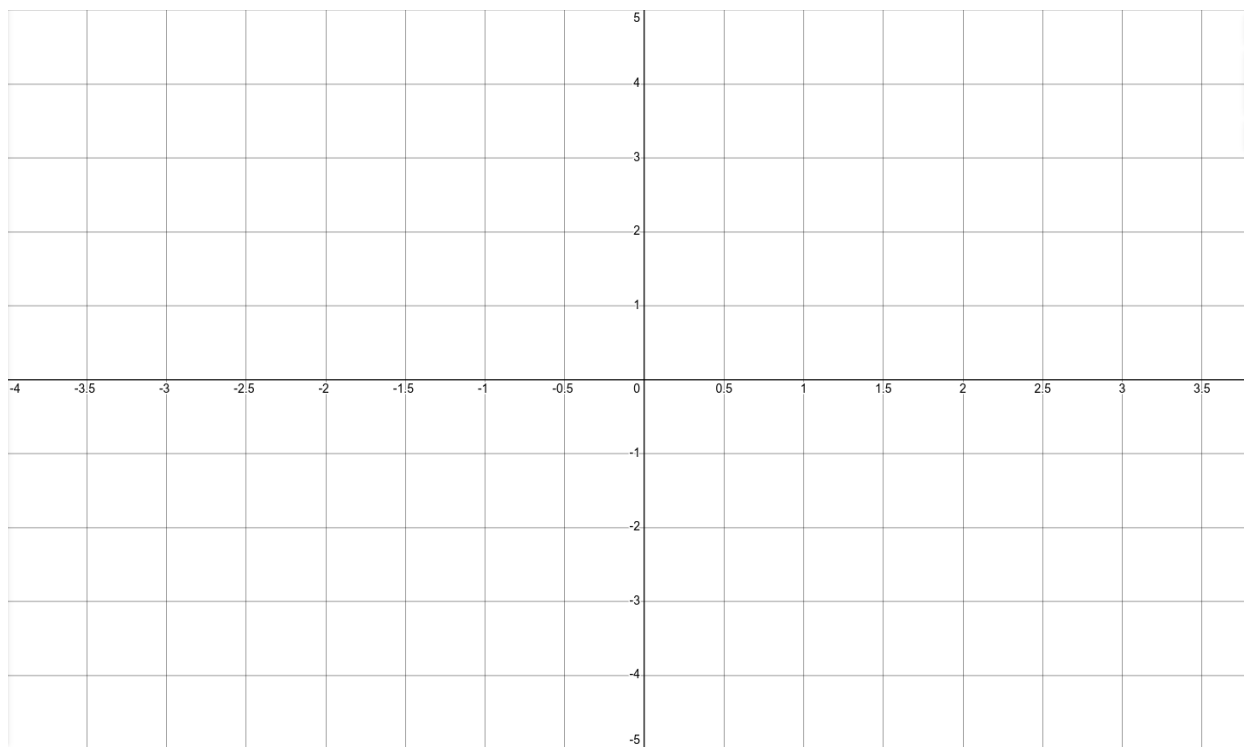
- (b) (4 points) Use part (a) to find the equation of the tangent line to the curve $y = f(x)$ at the point $(1, 1)$.

Question 5

The graph $y = f(x)$ is shown below.



(5 points) Sketch the graph of $f'(x)$ below, using the graph of $y = f(x)$.



Question 6

For each of the functions f below, compute $f'(x)$. Do not simplify your answer after applying the differentiation rules.

(a) (2 points) $f(x) = 3x^2 + 2x - 6$

(b) (5 points) $f(x) = \frac{2x^3 + 4x}{x - 2}$

(c) (5 points) $f(x) = 5\sqrt{x}(x^2 + 3x - 1)$

(d) (5 points) $f(x) = \sqrt[3]{x^2 + x + 1}$

Question 7

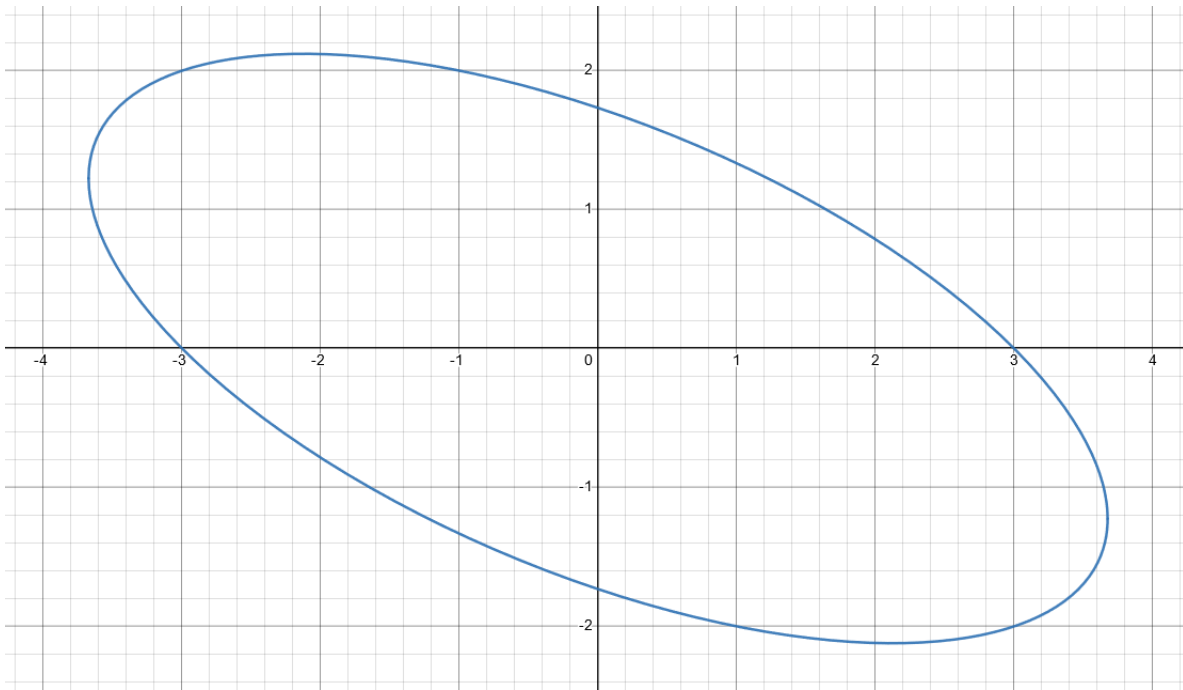
For each of the functions below, compute its derivative. Do not simplify your answer after applying the differentiation rules.

(a) (8 points) $f(t) = \frac{\sin(4t) + \tan(t)}{t^3 - 2}$

(b) (8 points) $g(x) = \sin((4x - 3)^2)$

Question 8

Consider the equation $x^2 + 2xy + 3y^2 = 9$, whose graph is the ellipse shown below.



(10 points) Find the equation of the tangent line to the curve at the point $(3, -2)$. Put your answer in the form $y = mx + b$.