Math 241 – Midterm 02

Fall 2022, 11/23/2022

4:00–5:15pm

Last name: ______________________________

First name: ______________________________

Section: ______________________________

Instructions:

- Write your last name, first name and section number above.
- Answer the eight questions on this exam.
- Show all the details of your work.
- No electronic devices are to be used during the exam (this includes calculators).
- The exam is closed book and closed notes.
- **Do not use L'Hôpital's rule anywhere on this exam**.
- You have 75 minutes to complete the exam.
- Turn in your exam when you are done or at the end of the 75-min period.

Sign below to acknowledge you have read the instructions.

Signature: ______________________________

May the Force be with you!

University of Hawai'i

Question 1 (10 pts)

If a spherical snowball melts so that its surface area decreases at a rate of $1\,\text{cm}^2/\text{min}$, find, using **Calculus**, the rate at which the diameter decreases when the diameter is 10 cm.

Note: The surface area of a sphere is $A = 4\pi r^2$.

Question 2 (10 pts)

Let $f(x) = \sqrt{4+x}$.

(a) (5 points) Find the linearization of the function f at the point $a = 0$.

(b) (5 points) Using the linearization, estimate the value of $\sqrt{4.1}$. Explain clearly how you obtained your answer and leave it in decimal form.

QUESTION 3 **(22 pts)**

Let $f(x) = \dfrac{3x^2 - 3}{x^2 + 3}$.

(a) (4 points) Using **Calculus**, find the vertical asymptotes (if any) and horizontal asymptotes (if any) of the function $f(x)$.

(b) (4 points) The first derivative of f is $f'(x) = \frac{24x}{(x^2+3)^2}$. Find the critical numbers (if any) and the open interval(s) of increase and decrease.

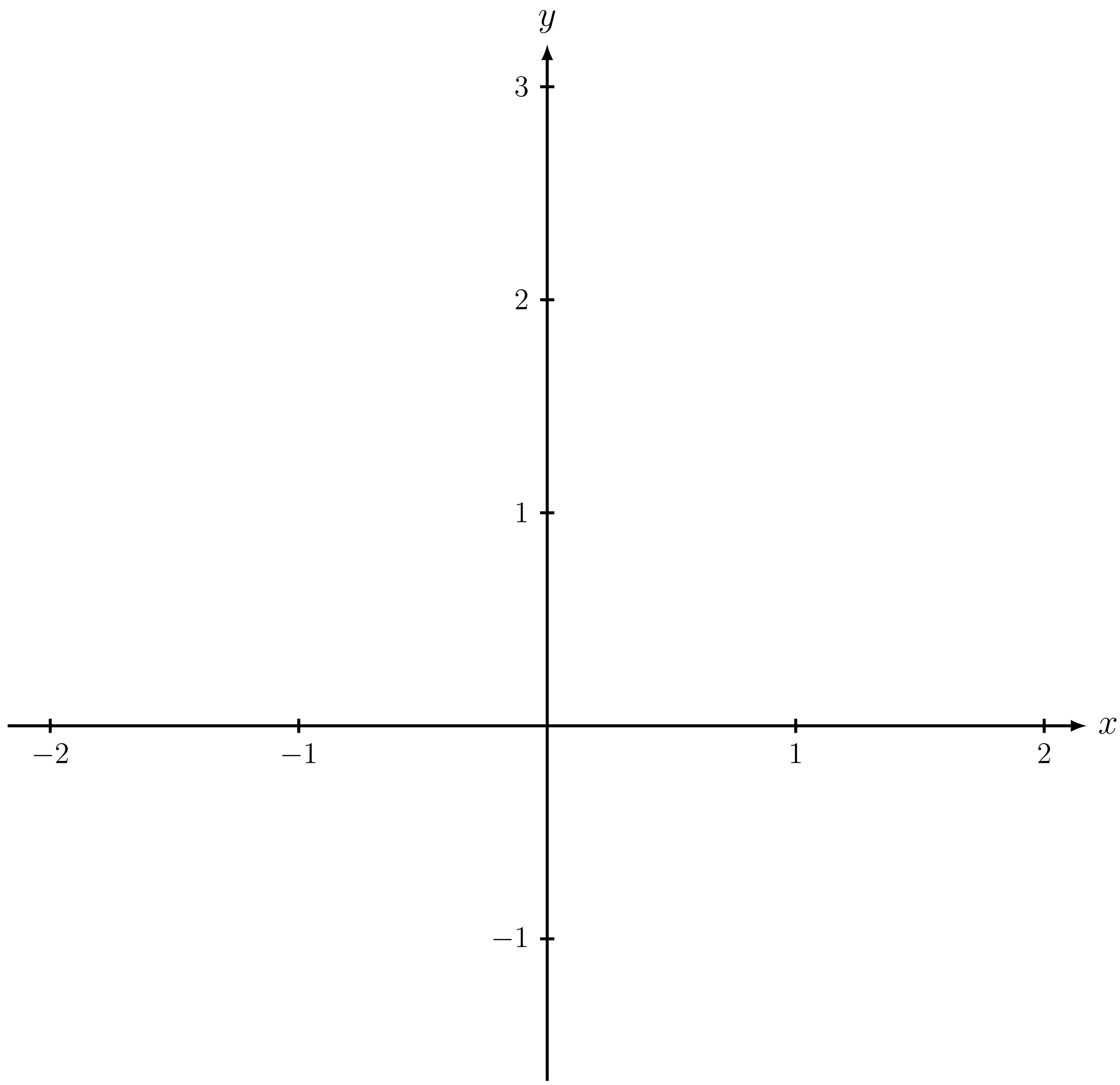
...Question 3 continued...

(c) (6 points) The second derivative of f is $f''(x) = \frac{-72(x^2-1)}{(x^2+3)^3}$. Find the x-coordinate of the inflection points (if any) and the open interval(s) of concavity.

(d) (4 points) Using one of the derivative tests, find the local maximum(s) and/or local minimum(s) of the function.

...Question 3 continued...

(e) (4 points) Sketch the graph of the function f in the axes below. Note that the y-intercept is -1 and the x-intercepts are $x = -1$ and $x = 1$.



Question 4 (10 pts)

Compute the following limits. If the limit does not exist, write explicitly DNE. Make sure to write all the details of your calculations.

(a) (5 points) $\lim_{x \to \infty} \dfrac{3x-2}{2x+1}$.

(b) (5 points) $\lim_{x \to -\infty} \dfrac{\sqrt{2x^2+1}}{3x-5}$.

QUESTION 5 **(15 pts)**

A box with an open top is to be constructed from a square piece of cardboard of side length 3 ft by cutting out a square from each of the four corners and bending up the sides. **Using calculus**, find the largest volume that such a box can have. Make sure to justify clearly your answer.

Question 6 **(15 pts)**

Find the most general antiderivative of the following functions.

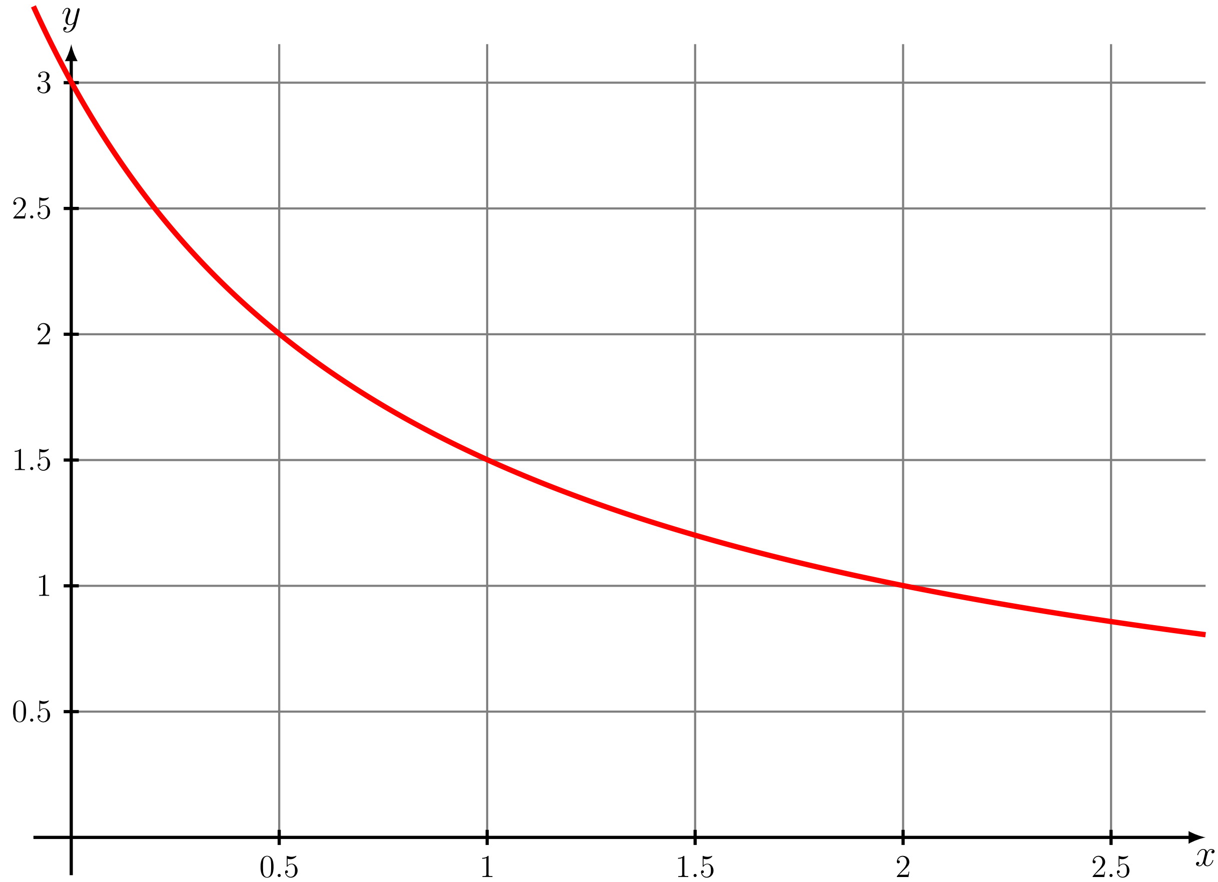
(a) (5 points) $f(x) = 4\sqrt{x} - 6x^2 + 3$.

(b) (5 points) $f(x) = \cos(x) + 2\sec^2(x)$.

(c) (5 points) $f(x) = x\sqrt{x} + \dfrac{x^2 + x}{x}$.

Question 7 **(8 pts)**

The graph of the function $f(x) = \dfrac{3}{1+x}$ is given below.



(a) (4 points) Estimate the area under the curve from $a = 0$ to $b = 2$ using two rectangles and right endpoints.

(b) (2 points) Draw the two rectangles from part (a) on the above picture of the graph of $f(x)$.

(c) (2 points) Is your answer over or under approximating the actual value of the area under the curve?

QUESTION 8 **(10 pts)**

Below is the recordings of the speed of a Kangoroo in Australia.

Time (s)	0	5	10	15	20	25	30	35	40
Speed (ft/s)	6	3	2	3	4	7	5	1	4

(a) (5 points) Using only the recordings at every 10 seconds, estimate the distance travelled after 40 seconds.

(b) (5 points) Does your estimate get better if you use instead the recordings at every 5 seconds?

Do not write on this page.

For official use only:

Question:	1	2	3	4	5	6	7	8	Total
Points:	10	10	22	10	15	15	8	10	100
Score:									