MATH-241
Sample MT-02

Created by Pierre-O. Parisé
2023/11/04, Spring 2023

Last name: _____

First name: _____

Section: _____

Instructions:

- Make sure to write your complete name on your copy.
- You must answer all eight (8) questions below and write your answers directly on the questionnaire.
- You have 75 minutes to complete the exam.
- When you are done (or at the end of the 75min period), return your copy.
- Devices such as smartphones, cellphones, laptops, tablets, e-readers, ipods, gameboys (and, you know, any other electronic devices that I haven't thought of) may not be used during the exam.
- You can not use a calculator.
- **Turn off your cellphones during the exam**.
- Lecture notes and the textbook are not allowed during the exam.
- You must show ALL your work to have full credit. An answer without justification is worth no points (except if it is mentioned explicitly in the question not to justify).
- Draw a square around your final answer.

Your Signature: _____

May the Force be with you!

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Question 1 **(10 pts)**

On the Big Island, it snows on the top of the Mauna Kea Volcano. You create a perfect spherical snowball with snow found on the Mauna Kea summit. While you go down the volcano, the air temperature increases and the snowball starts to melt. If it melts so that its surface area decreases at a rate of $2\text{cm}^2/\text{min}$, find the rate at which the volume decreases if the surface area is 10cm^2.

Note: The surface area of a sphere is $A = 4\pi r^2$ and the volume of a ball is $V = \frac{4}{3}\pi r^3$.

Question 2 **(10 pts)**

Let $f(x) = \sqrt{x+4}$.

(a) (5 points) Find the linearization of $f(x)$ at $a = 0$.

(b) (5 points) Use the linearization found in (a) to approximate $\sqrt{4.1}$.

Question 3 **(10 pts)**

(a) (5 points) Using Newton's method with $x_1 = 1$, find the approximation x_2 of the root of the equation

$$x^3 - x + 1 = 0.$$

(b) (5 points) For which starting points x_1 do Newton's method fail?

Question 4 **(20 pts)**

Use the guidelines learnt in lecture to sketch the curve $y = \frac{x}{x-1}$.

Question 5 **(15 pts)**

Find the value of the following limits.

(a) (5 points) $\lim_{x\to-\infty} \dfrac{x^3+4x+2}{10x^3+x^2+10}$.

(b) (5 points) $\lim_{x\to\infty} \dfrac{x+4}{\sqrt[3]{x^3+x+3}}$.

(c) (5 points) $\lim_{x\to-\infty} \dfrac{\sqrt{2x^2+2}}{x+4}$.

Question 6 **(15 pts)**

Answer the following:

(a) (5 points) Find the value of $\sum_{i=1}^{5}(2i)$.

(b) (5 points) Draw the region for which the area is described by the following expression:

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{1}{n}\left(1+\frac{i}{n}\right)^2.$$

(c) (5 points) Find all numbers c that satisfy the conclusion of the Mean Value Theorem for $f(x) = \frac{1}{x}$ on $[1, 3]$.

Question 7 **(10 pts)**

Find the dimensions of a rectangle with area 1000m^2 whose perimeter is as small as possible.

Question 8 **(10 pts)**

Let $f(x) = \sin x$.

(a) (5 points) Using **two** rectangles, approximate the area of the region between the x-axis and the graph of $f(x)$ between 0 and $\pi/2$.

(b) (5 points) Illustrate the region from part (a) and the approximating rectangles.

Do not write on this page.

For official use only:

Question:	1	2	3	4	5	6	7	8	Total
Points:	10	10	10	20	15	15	10	10	100
Score:									