Chapter 2

Derivatives

2.1 Derivatives and Rates of Change

4 Definition The **derivative of a function f at a number a**, denoted by $f'(a)$, is

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

if this limit exists.

Another notation:

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

EXAMPLE 4 Find the derivative of the function $f(x) = x^2 - 8x + 9$ at the number $a = 1$.

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$

$= f(1)$

$$= \lim_{h \to 0} \frac{(1+h)^2 - 8(1+h) + 9 - (2)}{h}$$

$$= \lim_{h \to 0} \frac{1 + 2h + h^2 - 8 - 8h + 7}{h}$$

$$= \lim_{h \to 0} \frac{-6h + h^2}{h}$$

$$= \lim_{h \to 0} \frac{(-6+h)h}{h}$$

$$= \lim_{h \to 0} (-6+h) = -6$$

So, $\boxed{f'(1) = -6}$

Example. Find the derivative at a = 3 of the function

$$f(x) = \frac{3}{x}.$$

$$f'(3) = \lim_{h \to 0} \frac{f(3+h) - f(3)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{3}{3+h} - 1}{h}$$

$$= \lim_{h \to 0} \frac{\left(\frac{3}{3+h} - 1\right)(3+h)}{h\,(3+h)}$$

$$= \lim_{h \to 0} \frac{3 - (3+h)}{h(3+h)}$$

$$= \lim_{h \to 0} \frac{-h}{h(3+h)}$$

$$= \lim_{h \to 0} \frac{-1}{3+h} = \boxed{-\frac{1}{3}}$$

$f'(3)$.

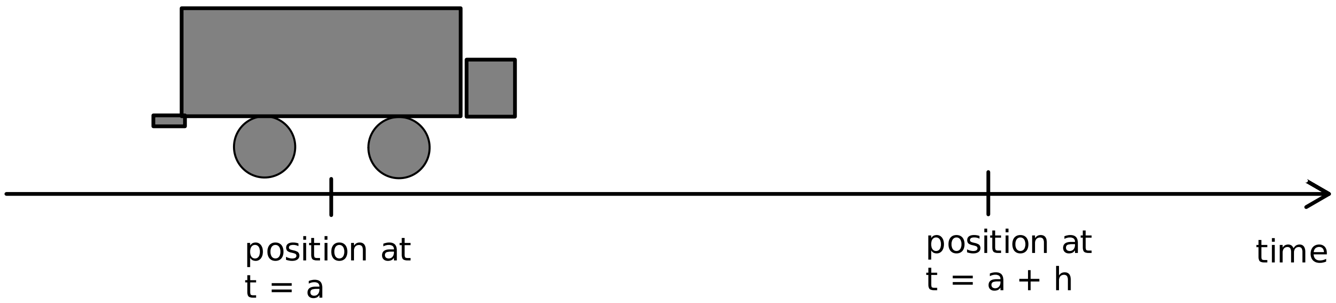
How do we find the tangent at a point P on a curve given by the graph of a function?

Answer:

The tangent line to $y = f(x)$ at $(a, f(a))$ is the line through $(a, f(a))$ whose slope is equal to $f'(a)$, the derivative of f at a.

$$y - f(a) = f'(a)(x - a)$$

Velocities



- Position at t: $s(t)$

- Average velocity from t = a to t = a + h: $\dfrac{s(a+h) - s(a)}{h}$

- Instantaneous Velocity at t = a:

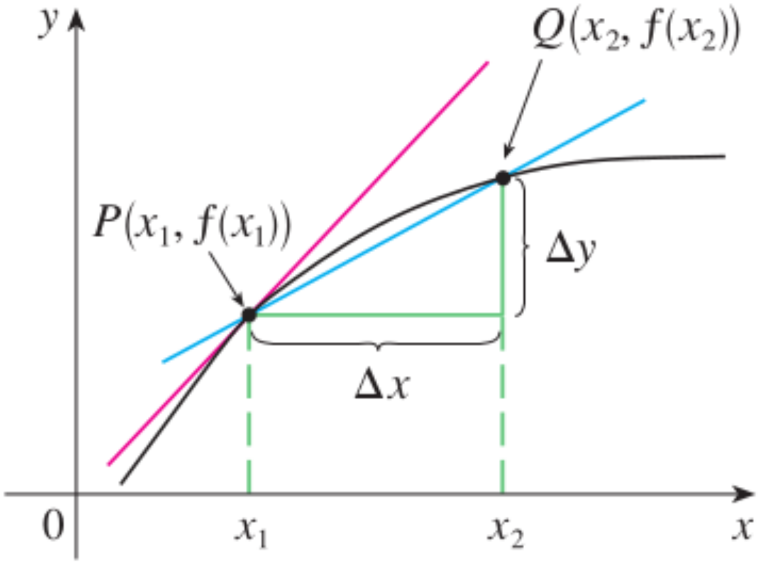
$$v(a) = \lim_{h \to 0} \frac{s(a+h) - s(a)}{h}$$

Rates of Change.

- Average rate of change when y varies by Δy and x varies by Δx :

$$\text{Average rate of change} = \frac{\Delta y}{\Delta x}$$

- Take limit as $\Delta x \to 0$



6 $$\text{instantaneous rate of change} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_2 \to x_1} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$