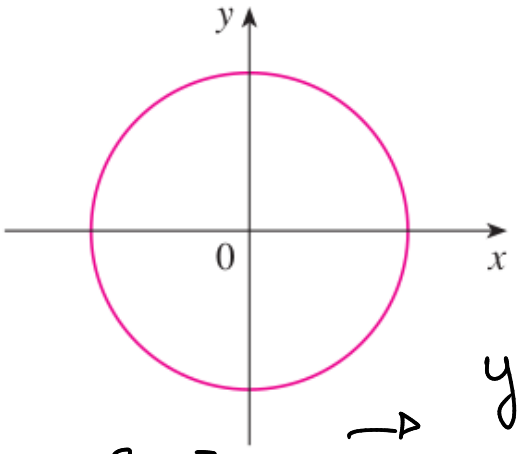
Chapter 2
Derivatives
2.6 Implicit Differentiation

Functions defined implicitly.

Geometry of curves.

circle



$x^2+y^2=1$ → $y=y(x)$ implicitly

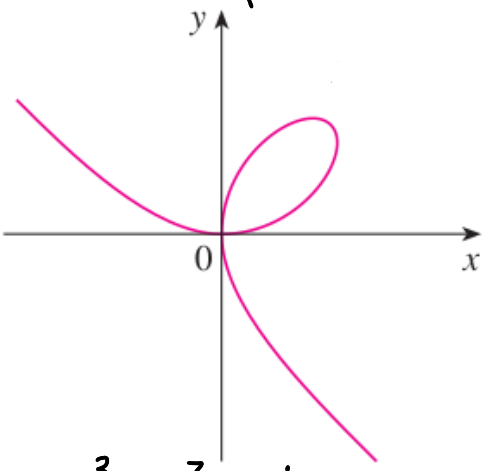
↳ $y^2=1-x^2$

↳ $y=\sqrt{1-x^2}$

& $y=-\sqrt{1-x^2}$

explicit.

Folium of Descartes



$x^3+y^3=6xy$

really hard to isolate y.

We think of y as $y=f(x)$

In Natural Science (Gas' Law).

$$\left(P+\frac{n^2a}{V^2}\right)(V-nb)=nRT$$

- P: Pressure
- V: Volume
- T: Temperature
- R, a, b are constants depending on the gas.

HOW DO WE FIND THE SLOPE/DERIVATIVE OF A FUNCTION $y=f(x)$ IF THE RULE IS GIVEN BY AN IMPLICIT EQUATION?

EXAMPLE 1

(a) If $x^2 + y^2 = 25$, find $\frac{dy}{dx}$.

(b) Find an equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$.

(a) $y = f(x)$.

① Take the derivative on both side

$$\frac{d}{dx}(x^2+y^2) = \frac{d}{dx}(25) \qquad \left[\frac{d}{dx}(\pi^2) = 0\right]$$

$$\Rightarrow \frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = 0$$

$$\Rightarrow 2x + 2y \cdot y' = 0$$

② Isolate y'

$$2y \cdot y' = -2x \Rightarrow y' = -\frac{x}{y}$$

(b) Eq. tangent line: $y - 4 = y'(3)(x-3)$

$$y'(3) = -\frac{3}{4} \Rightarrow y - 4 = \frac{-3}{4}(x-3)$$

$$\Rightarrow \boxed{y = -\frac{3x}{4} + \frac{25}{4}}$$

Main steps for implicit differentiation:

1) Take the derivative on each side of the relation.

2) Use the chain rule and other rules to make the computations.

3) Isolate the derivative dy/dx.

Example 2.

Let $x^3 + y^3 = 6xy$. Find the tangent line to the folium of Descartes at the point $(3, 3)$.

Desmos: https://www.desmos.com/calculator/efjuccxlrz

① $\frac{d}{dx}(x^3+y^3) = \frac{d}{dx}(6xy)$

$\Rightarrow 3x^2 + 3y^2 \cdot \frac{dy}{dx} = \frac{d}{dx}(6x)\, y + 6x \frac{dy}{dx}$

$\Rightarrow 3x^2 + 3y^2 \cdot \frac{dy}{dx} = 6y + 6x \frac{dy}{dx}$

② Replace x by 3 & y by 3

$27 + 27 \frac{dy}{dx} = 18 + 18 \frac{dy}{dx}$

$9 \frac{dy}{dx} = -9$

$\Rightarrow \left.\frac{dy}{dx}\right|_{x=3} = -1 \quad (= y'(3))$.

③ $y - 3 = y'(3)(x-3)$

$\Rightarrow y - 3 = -(x-3)$

$\Rightarrow \boxed{y = -x + 6}$