Math 241

Chapter 3

Section 3.5: Summary of Curve Sketching

Contents

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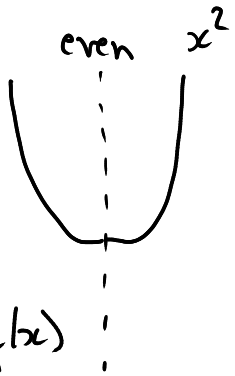
Example 1. Sketch the curve given by $y = \dfrac{2x^2}{x^2-1}$.

A. Domain: $x^2-1=0 \iff x = \pm 1$

$(-\infty,-1) \cup (-1,1) \cup (1,\infty)$.

B. y-intercept: $x=0 \Rightarrow y=0$.

x-intercept: $x^2=0 \iff x=0$.



C. (I) Even: $f(-x) = \dfrac{2(-x)^2}{(-x)^2-1} = \dfrac{2x^2}{x^2-1} = f(x)$

↳ So, function is even.

(II) Not odd. (III) No repetition.

D. (I) HA.

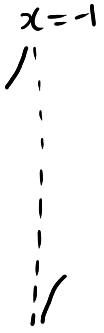
$\lim\limits_{x\to\infty} \dfrac{2x^2}{x^2-1} = \dfrac{2}{1} = 2 \to y=2$ is a HA.

$\lim\limits_{x\to-\infty} \dfrac{2x^2}{x^2-1} = \dfrac{2}{1} = 2 \to y=2$ is a HA.

(II) VA. $x=-1$

$\lim\limits_{x\to-1^-} \dfrac{2x^2}{(x-1)(x+1)} = \dfrac{2(-1)^2}{(-2)\,0^-} = +\infty$

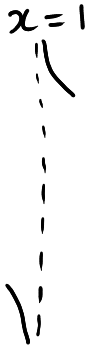
$\lim\limits_{x\to-1^+} \dfrac{2x^2}{(x-1)(x+1)} = \dfrac{2(-1)}{(-2)\,0^+} = -\infty$



$x=1$

$$\lim_{x\to 1^-} \frac{2x^2}{(x-1)(x+1)} = \frac{2}{0^- \cdot 2} = -\infty$$

$$\lim_{x\to 1^+} \frac{2x^2}{(x-1)(x+1)} = \frac{2}{0^+ \cdot 2} = +\infty$$



E. $f'(x) = \dfrac{-4x}{(x^2-1)^2} = \dfrac{-4x}{(x-1)^2(x+1)^2}$

$\Rightarrow$ $f'(x) = 0$ if $-4x = 0$ if $x = 0$.

and $f'(x)$ DNE if $x=-1$, $x=1$.

$\Rightarrow$ C.N. are $-1, 0, 1$.

$f''(x) = \dfrac{12x^4+4}{(x^2-1)^3} = \dfrac{12x^4+4}{(x-1)^3(x+1)^3}$ $\longrightarrow$ $\geq 4 > 0$.

$\Rightarrow$ $f''(x) = 0$ none.

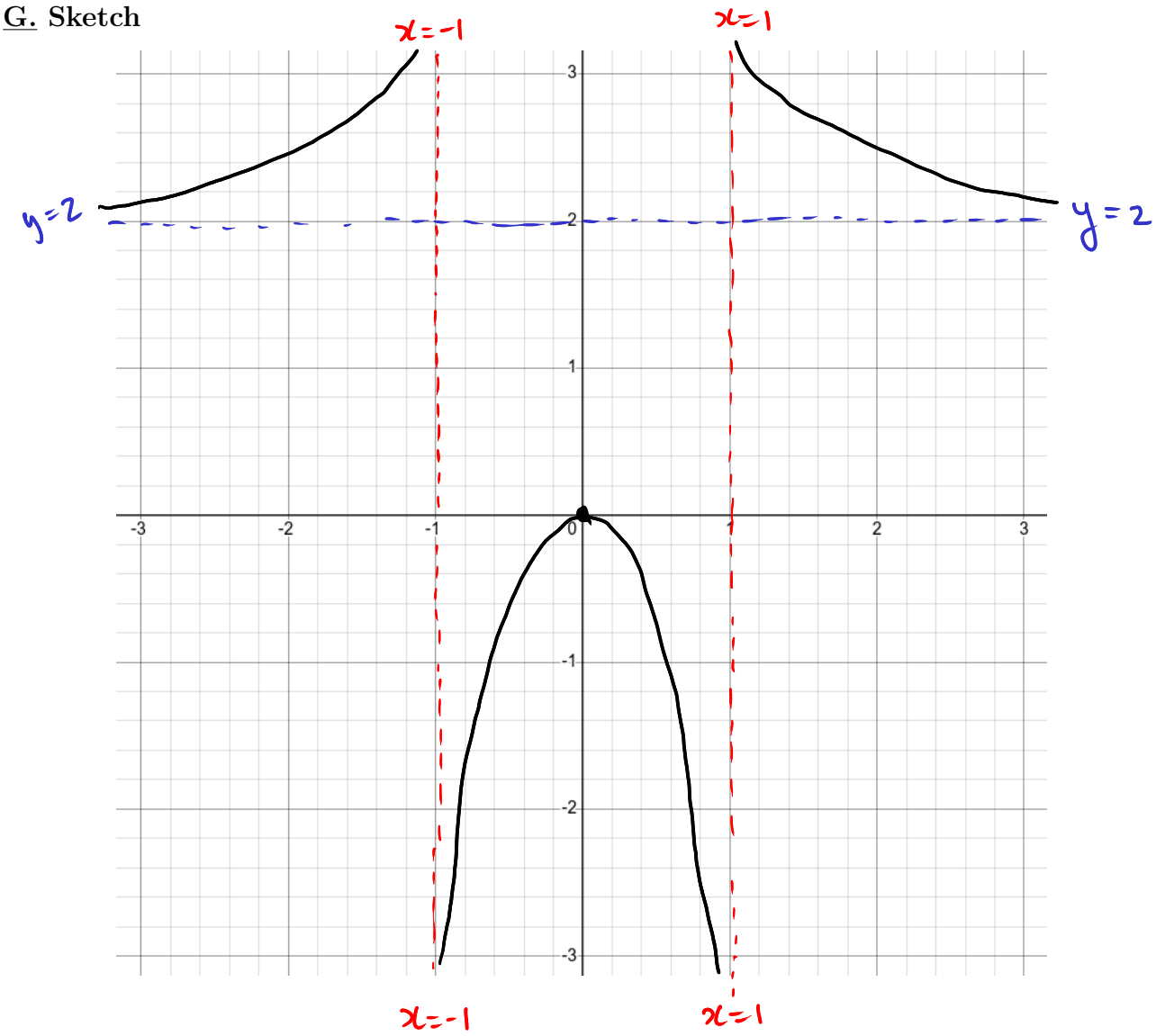
and $f''(x)$ DNE if $x=-1$, $x=1$

Possible I.P. are $x=-1$ and $x=1$.

F.

Factors	$x <$	-1	$< x <$	0	$< x <$	1	$< x$
$-4x$	$+$		$+$		$-$		$-$
$(x-1)^2$	$+$		$+$		$+$		$+$
$(x+1)^2$	$+$		$+$		$+$		$+$
$f'(x)$	$+$	DNE	$+$	0	$-$	DNE	$-$
$(x-1)^3$	$-$		$-$		$-$		$+$
$(x+1)^3$	$-$		$+$		$+$		$+$
$f''(x)$	$+$	DNE	$-$	-4	$-$		$+$
$f(x)$	↗	VA	↗	loc max	↘		↘

G. Sketch



Guideline For Sketching Curves

A. Find the domain of the function.

B. Find the y-intercept and x-intercept, that is $f(0)$ and when $f(x) = 0$.

C. Search for symmetries:

- **(I)** If $f(x) = f(-x)$ for all x, then the function is even.
- **(II)** If $-f(x) = f(-x)$ for all x, then the function is odd.
- **(III)** If $f(x+p) = f(x)$ for some p and all x, then the function repeats itself after a period p.

D. Find the asymptotes:

- **(I)** The horizontal asymptotes.
- **(II)** The vertical asymptotes.

E. Find the critical numbers and the possible points of inflections.

F. Construct the table:

- **(I)** Deduce the intervals of increase and decrease.
- **(II)** Deduce the intervals of concavity.
- **(III)** Deduce the local (global) maximum values and local (global) minimum values.

G. Sketch the graph of the functions.

DIY!

Example 2. Sketch the graph of $f(x) = \dfrac{x^2}{\sqrt{x+1}}$.