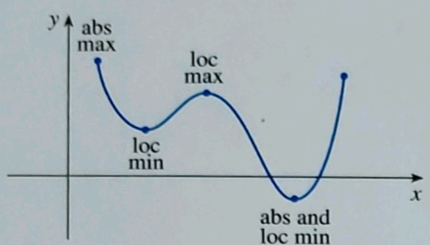
CHAPTER 3 CONCEPT CHECK ANSWERS

1. Explain the difference between an absolute maximum and a local maximum. Illustrate with a sketch.

The function value $f(c)$ is the absolute maximum value of f if $f(c)$ is the largest function value on the entire domain of f, whereas $f(c)$ is a local maximum value if it is the largest function value when x is near c.



2. What does the Extreme Value Theorem say?

If f is a continuous function on a closed interval $[a, b]$, then it always attains an absolute maximum and an absolute minimum value on that interval.

3. (a) State Fermat's Theorem.

If f has a local maximum or minimum at c, and if $f'(c)$ exists, then $f'(c) = 0$.

(b) Define a critical number of f.

A critical number of a function f is a number c in the domain of f such that either $f'(c) = 0$ or $f'(c)$ does not exist.

4. Explain how the Closed Interval Method works.

To find the absolute maximum and minimum values of a continuous function f on a closed interval $[a, b]$, we follow these three steps:

- Find the critical numbers of f in the interval (a, b) and compute the values of f at these numbers.
- Find the values of f at the endpoints of the interval.
- The largest of the values from the previous two steps is the absolute maximum value; the smallest of these values is the absolute minimum value.

5. (a) State Rolle's Theorem.

Let f be a function that satisfies the following three hypotheses:

- f is continuous on the closed interval $[a, b]$.
- f is differentiable on the open interval (a, b).
- $f(a) = f(b)$

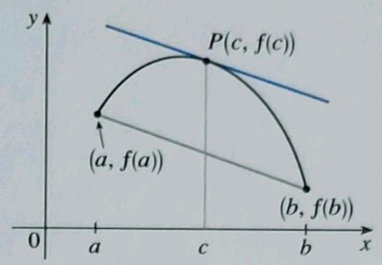
Then there is a number c in (a, b) such that $f'(c) = 0$.

(b) State the Mean Value Theorem and give a geometric interpretation.

If f is continuous on the interval $[a, b]$ and differentiable on (a, b), then there exists a number c between a and b such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

Geometrically, the theorem says that there is a point $P(c, f(c))$, where $a < c < b$, on the graph of f where the tangent line is parallel to the secant line that connects $(a, f(a))$ and $(b, f(b))$.



6. (a) State the Increasing/Decreasing Test.

If $f'(x) > 0$ on an interval, then f is increasing on that interval.

If $f'(x) < 0$ on an interval, then f is decreasing on that interval.

(b) What does it mean to say that f is concave upward on an interval I?

f is concave upward on an interval if the graph of f lies above all of its tangents on that interval.

(c) State the Concavity Test.

If $f''(x) > 0$ on an interval, then the graph of f is concave upward on that interval.

If $f''(x) < 0$ on an interval, then the graph of f is concave downward on that interval.

(d) What are inflection points? How do you find them?

Inflection points on the graph of a continuous function f are points where the curve changes from concave upward to concave downward or from concave downward to concave upward. They can be found by determining the values at which the second derivative changes sign.

7. (a) State the First Derivative Test.

Suppose that c is a critical number of a continuous function f.

- If f' changes from positive to negative at c, then f has a local maximum at c.
- If f' changes from negative to positive at c, then f has a local minimum at c.
- If f' is positive to the left and right of c, or negative to the left and right of c, then f has no local maximum or minimum at c.

(b) State the Second Derivative Test.

Suppose f'' is continuous near c.

- If $f'(c) = 0$ and $f''(c) > 0$, then f has a local minimum at c.
- If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at c.

(continued)

(c) What are the relative advantages and disadvantages of these tests?

The Second Derivative Test is sometimes easier to use, but it is inconclusive when $f''(c) = 0$ and fails if $f''(c)$ does not exist. In either case the First Derivative Test must be used.

8. Explain the meaning of each of the following statements.

(a) $\lim_{x \to \infty} f(x) = L$ means that the values of $f(x)$ can be made arbitrarily close to L by requiring x to be sufficiently large.

(b) $\lim_{x \to -\infty} f(x) = L$ means that the values of $f(x)$ can be made arbitrarily close to L by requiring x to be sufficiently large negative.

(c) $\lim_{x \to \infty} f(x) = \infty$ means that the values of $f(x)$ can be made arbitrarily large by requiring x to be sufficiently large.

(d) The curve $y = f(x)$ has the horizontal asymptote $y = L$.

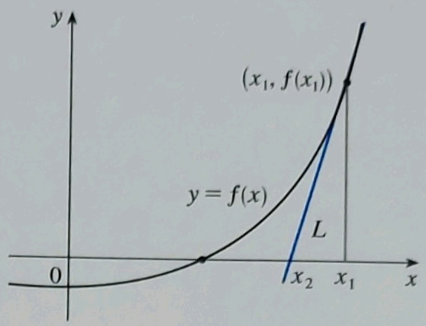
The line $y = L$ is called a horizontal asymptote of the curve $y = f(x)$ if either $\lim_{x \to \infty} f(x) = L$ or $\lim_{x \to -\infty} f(x) = L$.

9. If you have a graphing calculator or computer, why do you need calculus to graph a function?

Calculus reveals all the important aspects of a graph, such as local extreme values and inflection points, that can be missed when relying solely on technology. In many cases we can find exact locations of these key points rather than approximations. Using derivatives to identify the behavior of the graph also helps us choose an appropriate viewing window and alerts us to where we may wish to zoom in on a graph.

10. (a) Given an initial approximation x_1 to a root of the equation $f(x) = 0$, explain geometrically, with a diagram, how the second approximation x_2 in Newton's method is obtained.

We find the tangent line L to the graph of $y = f(x)$ at the point $(x_1, f(x_1))$. Then x_2 is the x-intercept of L.



(b) Write an expression for x_2 in terms of x_1, $f(x_1)$, and $f'(x_1)$.

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

(c) Write an expression for x_{n+1} in terms of x_n, $f(x_n)$, and $f'(x_n)$.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

(d) Under what circumstances is Newton's method likely to fail or to work very slowly?

Newton's method is likely to fail or to work very slowly when $f'(x_1)$ is close to 0. It also fails when $f'(x_i)$ is undefined.

11. (a) What is an antiderivative of a function f?

A function F is an antiderivative of f if $F'(x) = f(x)$.

(b) Suppose F_1 and F_2 are both antiderivatives of f on an interval I. How are F_1 and F_2 related?

They are identical or they differ by a constant.