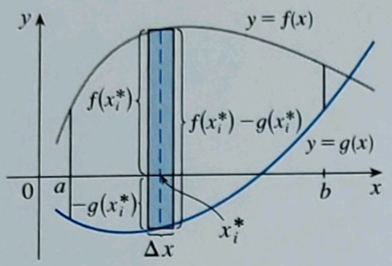
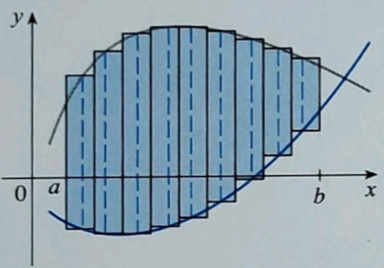
CHAPTER 5 CONCEPT CHECK ANSWERS

1. (a) Draw two typical curves $y = f(x)$ and $y = g(x)$, where $f(x) \geq g(x)$ for $a \leq x \leq b$. Show how to approximate the area between these curves by a Riemann sum and sketch the corresponding approximating rectangles. Then write an expression for the exact area.



A Riemann sum that approximates the area between these curves is $\sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)]\,\Delta x$. A sketch of the corresponding approximating rectangles:

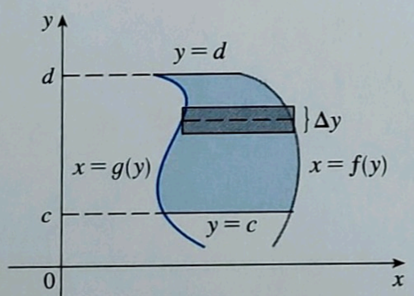


An expression for the exact area is

$$\lim_{n \to \infty} \sum_{i=1}^{n} [f(x_i^*) - g(x_i^*)]\,\Delta x = \int_a^b [f(x) - g(x)]\,dx$$

(b) Explain how the situation changes if the curves have equations $x = f(y)$ and $x = g(y)$, where $f(y) \geq g(y)$ for $c \leq y \leq d$.

Instead of using "top minus bottom" and integrating from left to right, we use "right minus left" and integrate from bottom to top: $A = \int_c^d [f(y) - g(y)]\,dy$



2. Suppose that Sue runs faster than Kathy throughout a 1500-meter race. What is the physical meaning of the area between their velocity curves for the first minute of the race?

It represents the number of meters by which Sue is ahead of Kathy after 1 minute.

3. (a) Suppose S is a solid with known cross-sectional areas. Explain how to approximate the volume of S by a Riemann sum. Then write an expression for the exact volume.

We slice S into n "slabs" of equal width Δx. The volume of the ith slab is approximately $A(x_i^*)\,\Delta x$, where x_i^* is a sample point in the ith slab and $A(x_i^*)$ is the cross-sectional area of S at x_i^*. Then the volume of S is approximately $\sum_{i=1}^{n} A(x_i^*)\,\Delta x$ and the exact volume is

$$V = \lim_{n \to \infty} \sum_{i=1}^{n} A(x_i^*)\,\Delta x = \int_a^b A(x)\,dx$$

(b) If S is a solid of revolution, how do you find the cross-sectional areas?

If the cross-section is a disk, find the radius in terms of x or y and use $A = \pi(\text{radius})^2$. If the cross-section is a washer, find the inner radius r_{in} and outer radius r_{out} and use $A = \pi(r_{\text{out}}^2) - \pi(r_{\text{in}}^2)$.

4. (a) What is the volume of a cylindrical shell?

$$V = 2\pi rh\,\Delta r = (\text{circumference})(\text{height})(\text{thickness})$$

(b) Explain how to use cylindrical shells to find the volume of a solid of revolution.

We approximate the region to be revolved by rectangles, oriented so that revolution forms cylindrical shells rather than disks or washers. For a typical shell, find the circumference and height in terms of x or y and calculate

$$V = \int_a^b (\text{circumference})(\text{height})(dx \text{ or } dy)$$

(c) Why might you want to use the shell method instead of slicing?

Sometimes slicing produces washers or disks whose radii are difficult (or impossible) to find explicitly. On other occasions, the cylindrical shell method leads to an easier integral than slicing does.

5. Suppose that you push a book across a 6-meter-long table by exerting a force $f(x)$ at each point from $x = 0$ to $x = 6$. What does $\int_0^6 f(x)\,dx$ represent? If $f(x)$ is measured in newtons, what are the units for the integral?

$\int_0^6 f(x)\,dx$ represents the amount of work done. Its units are newton-meters, or joules.

6. (a) What is the average value of a function f on an interval $[a, b]$?

$$f_{\text{ave}} = \frac{1}{b - a}\int_a^b f(x)\,dx$$

(b) What does the Mean Value Theorem for Integrals say? What is its geometric interpretation?

If f is continuous on $[a, b]$, then there is a number c in $[a, b]$ at which the value of f is exactly equal to the average value of the function, that is, $f(c) = f_{\text{ave}}$. This means that for positive functions f, there is a number c such that the rectangle with base $[a, b]$ and height $f(c)$ has the same area as the region under the graph of f from a to b.