Chapter 2

Derivatives

2.2 The Derivatives as a Function

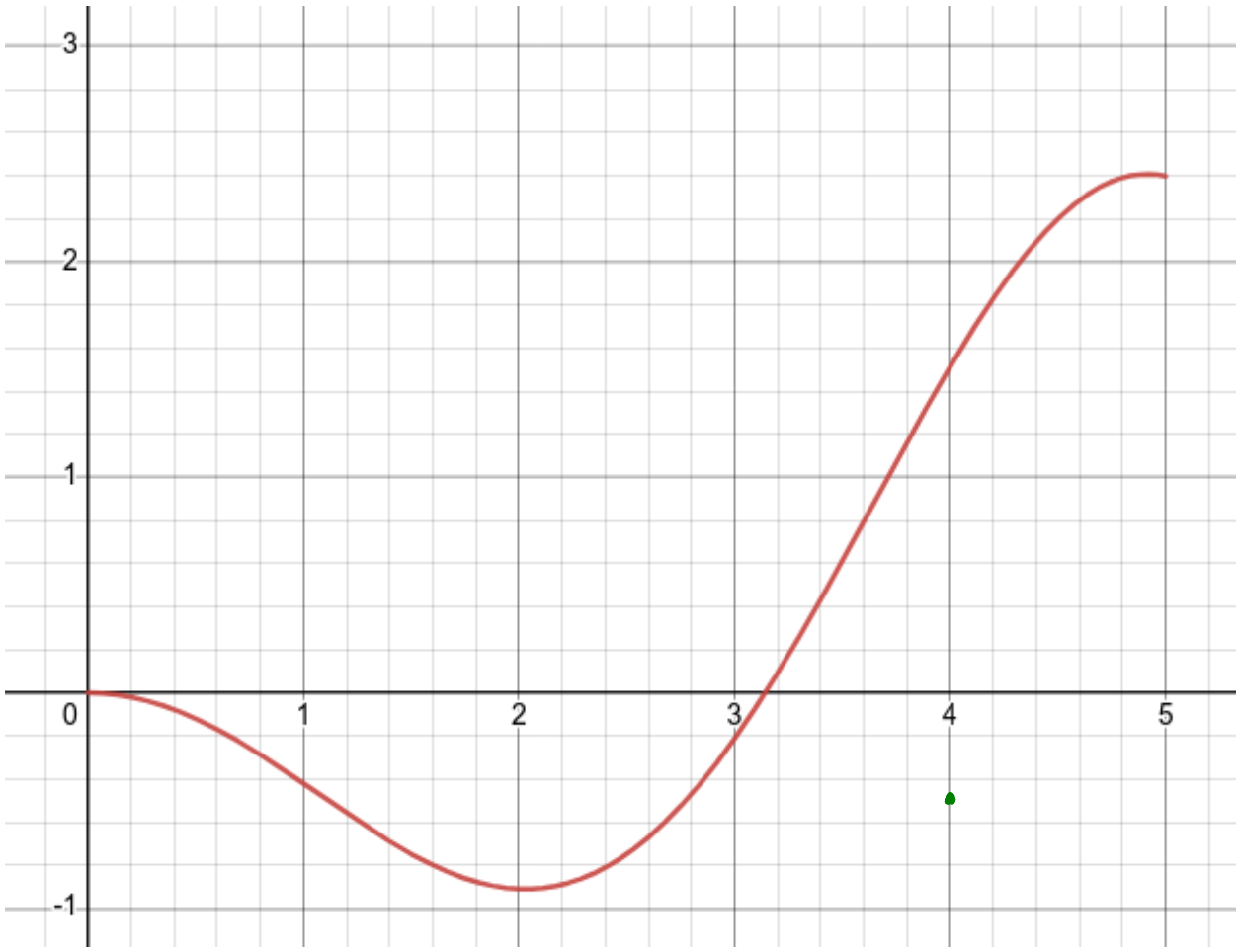
The derivative as a function.

$$f'(x) = \lim_{h \to 0} \frac{f(x + h) - f(x)}{h}$$

Dom of f':

EXAMPLE 1 The graph of a function f is given . Use it to sketch the graph of the derivative f'.

Desmos: https://www.desmos.com/calculator/o7lfvk2sar



EXAMPLE 3 If $f(x) = \sqrt{x}$, find the derivative of f. State the domain of f'.

(b) Illustrate this formula by comparing the graphs of f and f'. (Do it with Desmos)

Other notations for the derivative.

$$f'(x) = y' = \frac{dy}{dx} = \frac{df}{dx} = \frac{d}{dx}f(x) = Df(x) = D_x f(x)$$

EXAMPLE. What is the value of $\left.\frac{dy}{dx}\right|_{x=2}$ if $y = x^2$.

Leibniz Notation: $\left.\frac{dy}{dx}\right|_{x=a} = f'(a).$

3 Definition A function f is **differentiable at** a if $f'(a)$ exists. It is **differentiable on an open interval** (a, b) [or (a, ∞) or $(-\infty, a)$ or $(-\infty, \infty)$] if it is differentiable at every number in the interval.

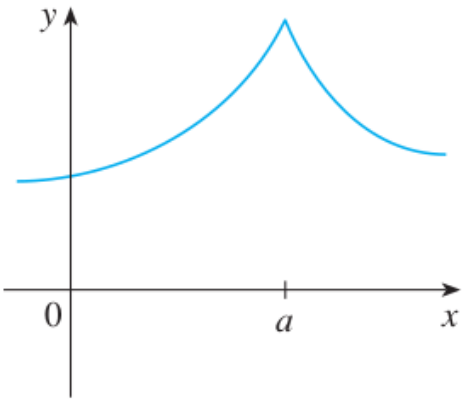
EXAMPLE 5 Where is the function $f(x) = |x|$ differentiable?

Important Result:

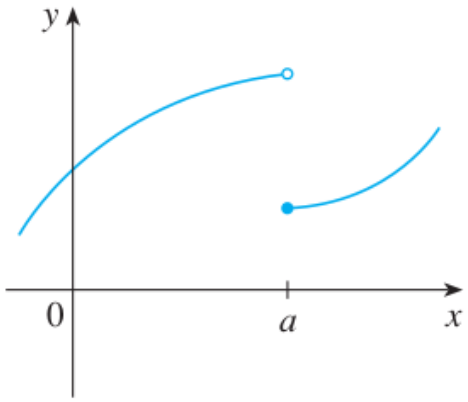
4 Theorem If f is differentiable at a, then f is continuous at a.

Remark:

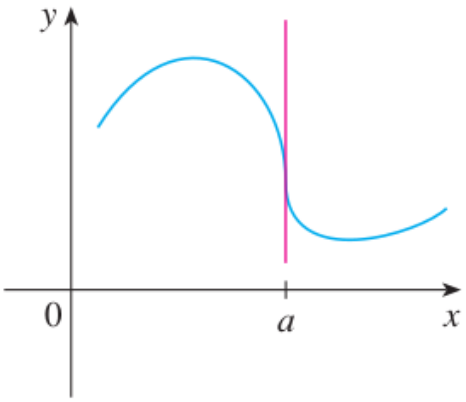
How can a Function Fail to be diffentiable?



(a) A corner



(b) A discontinuity



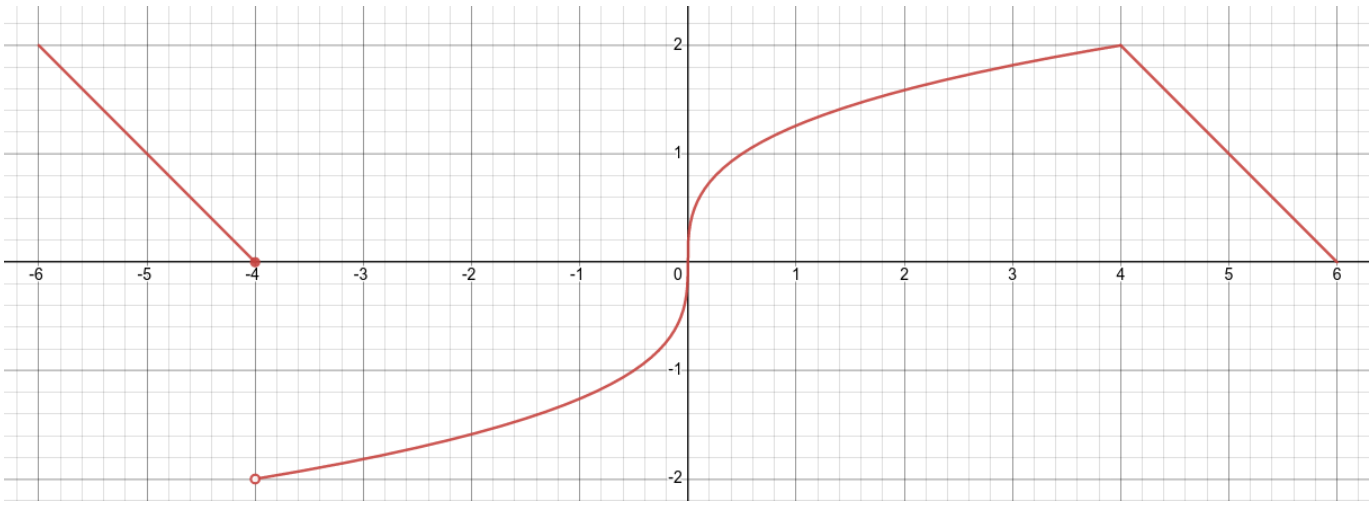
(c) A vertical tangent

(a)

(b)

(c)

Example. The graph of the function is given. State, with reasons, the numbers at which the function is NOT differentiable.
Desmos: https://www.desmos.com/calculator/d0aztxzxta



Higher Derivatives.

Second derivative:

$$\underbrace{\frac{d}{dx}}_{\text{derivative of}}\underbrace{\left(\frac{dy}{dx}\right)}_{\text{first derivative}} = \underbrace{\frac{d^2y}{dx^2}}_{\text{second derivative}}$$

Other notations:

EXAMPLE 6 If $f(x) = x^3 - x$, find and interpret $f''(x)$.

Acceleration:

Third Derivative.

$$y''' = f'''(x) = \frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3}$$

Jerk: $j = \frac{da}{dt} = \frac{d^3s}{dt^3}$

n-th Derivative.

$$y^{(n)} = f^{(n)}(x) = \frac{d^n y}{dx^n}$$

EXAMPLE 7 If $f(x) = x^3 - x$, find $f'''(x)$ and $f^{(4)}(x)$.