

# Chapter 2

## Derivatives

2.5 Chain Rule

How do you differentiate the function  $F(x) = \sqrt{x^2 + 1}$  ?

**The Chain Rule** If  $g$  is differentiable at  $x$  and  $f$  is differentiable at  $g(x)$ , then the composite function  $F = f \circ g$  defined by  $F(x) = f(g(x))$  is differentiable at  $x$  and  $F'$  is given by the product

$$F'(x) = f'(g(x)) \cdot g'(x)$$

In Leibniz notation, if  $y = f(u)$  and  $u = g(x)$  are both differentiable functions, then

$$\frac{dy}{dx} = \frac{dy}{du} \frac{du}{dx}$$

Main idea:

$$\frac{d}{dx} \underbrace{f}_{\text{outer function}} \underbrace{(g(x))}_{\text{evaluated at inner function}} = \underbrace{f'}_{\text{derivative of outer function}} \underbrace{(g(x))}_{\text{evaluated at inner function}} \cdot \underbrace{g'(x)}_{\text{derivative of inner function}}$$

**EXAMPLE 2** Differentiate (a)  $y = \sin(x^2)$  and (b)  $y = \sin^2 x$ .

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**EXAMPLE 4** Find  $f'(x)$  if  $f(x) = \frac{1}{\sqrt[3]{x^2 + x + 1}}$ .

**EXAMPLE 6** Differentiate  $y = (2x + 1)^5(x^3 - x + 1)^4$ .