Chapter 3
Applications of Derivatives

3.8 Newton's Method

Roots of polynomials.

- for quadratic polynomial $f(x) = ax^2 + bx + c$, the roots are given by:

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{a}$$

- There are formulas for cubics and quartics (horribly long...).
  -> cubic: https://encyclopediaofmath.org/wiki/Cardano_formula
  -> quartic: https://encyclopediaofmath.org/wiki/Ferrari_method

- For polynomials of degree greater than 4, there is no general formula!
  Interested in the proof: http://www.math.caltech.edu/~jimlb/abel.pdf

Niels Henrik Abel

- 1802-1829
- Died from Turberculosis

Evariste Galois

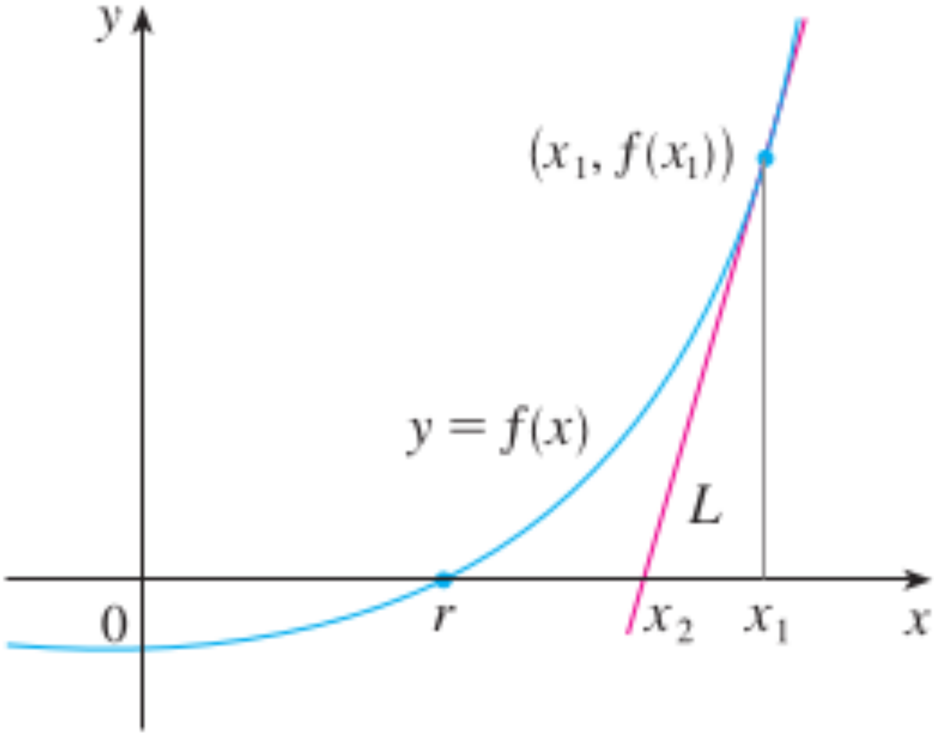
- Died in a duel for a mysterious mistress...
- 1811-1832

The urgent need of Newton's method!

KEY IDEAS:

- The tangent line approximate well the function.
- Replace the fonction with its tangent line.
- Intersect the tangent line with the x-axis.

Data:



$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Example. Starting with $x_1 = 2$, find the third approximation x_3 to the root of the equation $\dfrac{x^3}{2} - 3x = 0$.

n	xn
1	2
2	2.666666667
3	2.473429952
4	2.44983289
5	2.449489815
6	2.449489743
7	2.449489743
8	2.449489743
9	2.449489743
10	2.449489743
11	2.449489743
12	2.449489743
13	2.449489743
14	2.449489743
15	2.449489743

Newton's Method May Fail

Take a look at the formula in Newton's Method:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Where do you think this formula might fail?

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Example. Redo the last example with $x_1 = -1.14$.

Desmos: https://www.desmos.com/calculator/nm3bpdg95t

$MANY^{MANY} APPLICATIONS$!!!

- Finding solutions to general equations such as

$$\cos(x) = x$$

- At the core of many numerical methods in ingeneering.

- Generate wonderful fractal pictures:
  Watch the 3blue1brown video
  https://www.youtube.com/watch?v=-RdOwhmqP5s