MATH 241

Chapter 3

SECTION 3.9: ANTIDERIVATIVES

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DEFINITION

A function F is an **antiderivative** of a function f if F'(x) = f(x).

EXAMPLE 1. Find an antiderivative of the following functions.

(a)
$$f(x) = x^2$$
.

(b)
$$g(x) = 3x^3 + \cos(x)$$
.

(b)
$$g(x) = 3x^3 + \cos(x)$$
. **(c)** $h(x) = x^{2/3} + 4\sec^2(x)$.

Remarks:

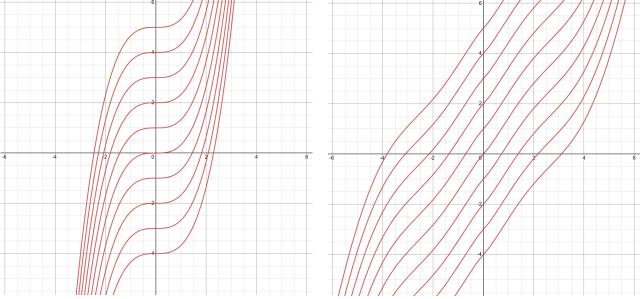
- Recall that f'(x) = g'(x) if and only if f(x) = g(x) + C for some constant C.
- There are more than just one antiderivative!

GENERAL ANTIDERIVATIVES

The most general antiderivative of a function f is

$$F(x) + C$$

where C is a constant.



- (a) Several Antiderivatives of $f(x) = x^2$, that is $\frac{x^3}{3} + C$
 - (b) Several antiderivatives of $f(x) = x^{2/3} + \cos(x)$, that is $\frac{3}{5}x^{5/3} + \sin(x) + C$.

EXAMPLE 2. Find the most general antiderivative of each of the following functions.

(a)
$$f(x) = \sin x$$
.

(b)
$$f(x) = x^n, n \ge 0.$$

Function	Particular antiderivative	Function	Particular antiderivative
cf(x) f(x) + g(x)	cF(x) $F(x) + G(x)$	$\cos x$ $\sin x$	$\sin x$ $-\cos x$
$x^n (n \neq -1)$	$\frac{x^{n+1}}{n+1}$	$\sec^2 x$ $\sec x \tan x$	tan x sec x

Figure 2: Properties and some Antiderivatives

EXAMPLE 3. Find all functions g such that

$$g'(x) = 4\sin x + \frac{2x^5 - \sqrt{x}}{x}.$$

INITIAL CONDITION

EXAMPLE 4. Find F if $F'(x) = x\sqrt{x}$ and F(1) = 2.

EXAMPLE 5. Find F if $F'(x) = \frac{1}{x^2}$ and F(1) = 2.