

Math 241/251A Final, Fall 2022

December 14, 2022, 12:00-2:00

Name: _____

Draw a circle around your section number below.

	Instructor	TA	Recitation	Question	Points	Score
1	Hugh Chou	Katie Menssen	W 10:30-11:20	1	16	
2	Hugh Chou	Katie Menssen	W 1:30-2:20	2	6	
3	Vasu Tewari	Dennis Le	Th 8:30-9:20	3	8	
4	Vasu Tewari	Dennis Le	Th 10:30-11:20	4	16	
5	Lyon Lanerolle	Drew Polakowski	F 9:30-10:20	5	8	
6	Lyon Lanerolle	Katie Menssen	F 12:30-1:20	6	5	
7	Julian Hachmeister	Julian Hachmeister	Th 10:30-11:20	7	6	
8	Pierre-Olivier Parise	Sydney Fields	Tu 10:30-11:20	8	10	
9	Pierre-Olivier Parise	Sydney Fields	Tu 1:30-2:20	9	10	
10	Nicolas Antin	Janitha Aswedige	W 10:30-11:20	10	15	
11	Nicolas Antin	Janitha Aswedige	W 1:30-2:20	11	15	
12	Ryan Sasaki	Janitha Aswedige	F 9:30-10:20	12	12	
13	Ryan Sasaki	Sydney Fields	F 1:30-2:20	13	10	
14	Monir Zaman	Dennis Le	Th 9:30-10:20	14	13	
15	Monir Zaman	Samuel Miller	Th 12:30-1:20	Total:	150	
251A	Thomas Hangelbroek	Rico Vicente	F 8:30-9:20			

- You may not use notes or calculators on the test.
- You may not use electronic devices or access the internet.
- Please ask if anything seems confusing or ambiguous.
- You must show all your work and make clear what your final solution is (e.g. by drawing a box around it).
- You have 2 hours to complete this exam.
- Good luck!

1. Calculate the following limits. Do **not** use l'Hôpital's rule. If the limit is infinite, specify whether it is $+\infty$ or $-\infty$.

(a) (4 points) $\lim_{x \rightarrow 4} \frac{x^2 - 4x}{x^2 - 3x - 4}$

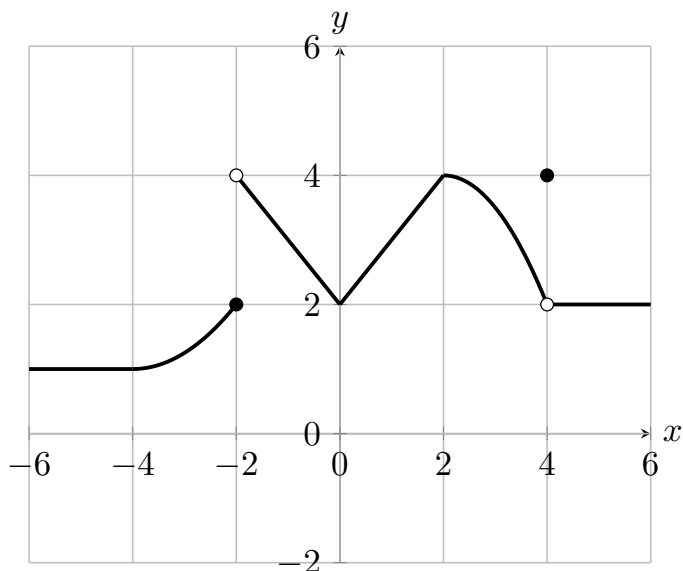
(b) (4 points) $\lim_{x \rightarrow 3^-} \frac{x^2 - 9}{|x - 3|}$

(c) (4 points) $\lim_{x \rightarrow 0} \frac{x}{5x^2 + 6 \sin x}$

(d) (4 points) $\lim_{x \rightarrow \infty} \frac{3x + 1}{\sqrt{2x^2 + \cos x}}$

2. (6 points) Let $f(x) = \frac{1}{x^2}$. Using the definition of the derivative as a limit, calculate $f'(3)$.
(Warning: You will not get credit if you use the rules of differentiation.)

3. Answer the following questions about the function $f(x)$ whose graph is depicted below. You do **not** need to justify your answers.



- (a) (1 point) What is the value of $\lim_{x \rightarrow -2^+} f(x)$?
- (b) (1 point) What is the value of $\lim_{x \rightarrow -2^-} f(x)$?
- (c) (2 points) For which value(s) of a does $\lim_{x \rightarrow a} f(x)$ fail to exist? If there are none, write “none”.
- (d) (2 points) For which value(s) of a is f discontinuous at $x = a$? If there are none, write “none”.
- (e) (2 points) For which value(s) of a does $f'(a)$ fail to exist? If there are none, write “none”.

4. Differentiate the following functions.

(a) (4 points) $\frac{x^4}{x^3 + 5x}$

(b) (4 points) $x^3 \cos(2x + 1)$

(c) (4 points) $(\tan x - 4x^2 + 3\sqrt{x})^9$

(d) (4 points) $\int_3^{x^2+1} \frac{dt}{\sqrt{4 + \sin t}}$

5. (8 points) Find an equation for the line tangent to the curve $x^2y + xy^3 + 42 = 0$ at the point $(-2, 3)$.

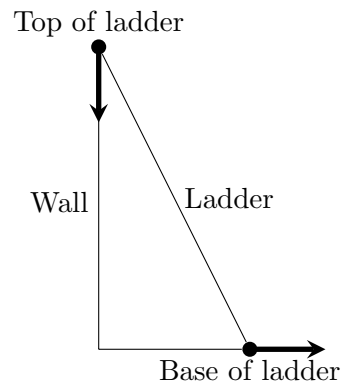
6. (5 points) Use linear approximation and the fact that $\sqrt[3]{8} = 2$ to estimate the value of $\sqrt[3]{8.12}$. Express your final answer as either a **fraction** in lowest terms or as a **decimal number** with two digits after the decimal point.

7. Consider the equation $2x^5 + 4x^3 = 1$.

(a) (3 points) Explain why this equation must have a solution in the interval $(0, 1)$. State any theorem(s) you use in your explanation.

(b) (3 points) Use Rolle's theorem to explain why this equation cannot have more than one solution in the interval $(0, 1)$.

8. (10 points) A ladder that is 5 meters long is leaning against a wall. Its base begins to slide horizontally away from the wall at a constant rate of 6 meters per second. Assuming that the top of the ladder slides vertically downward and always touches the wall, how quickly is the top of the ladder descending when its base is 3 meters away from the wall?



9. (10 points) A company that sells canned soup wants to design a cylindrical can that encloses as much soup as possible using $24\pi \text{ cm}^2$ of metal. What is the maximum volume that such a cylinder (i.e., a cylinder whose surface area is $24\pi \text{ cm}^2$) can enclose? (Recall that the volume of a cylinder of radius r and height h is $\pi r^2 h$, and the surface area is $2\pi r h + 2\pi r^2$.)

10. Consider the function $f(x) = \frac{x+1}{2x+3}$. The first and second derivatives of f are $f'(x) = \frac{1}{(2x+3)^2}$ and $f''(x) = \frac{-4}{(2x+3)^3}$, respectively. Answer the following questions about f .

(Note: This problem continues on the next page and the page thereafter.)

(a) (2 points) On which open interval(s) is f increasing? If there are none, write “none”.

(b) (2 points) On which open interval(s) is f decreasing? If there are none, write “none”.

(c) (2 points) On which open interval(s) is f concave up? If there are none, write “none”.

$\left(\text{Problem 10, continued. Recall that } f(x) = \frac{x+1}{2x+3}, f'(x) = \frac{1}{(2x+3)^2}, f''(x) = \frac{-4}{(2x+3)^3}. \right)$

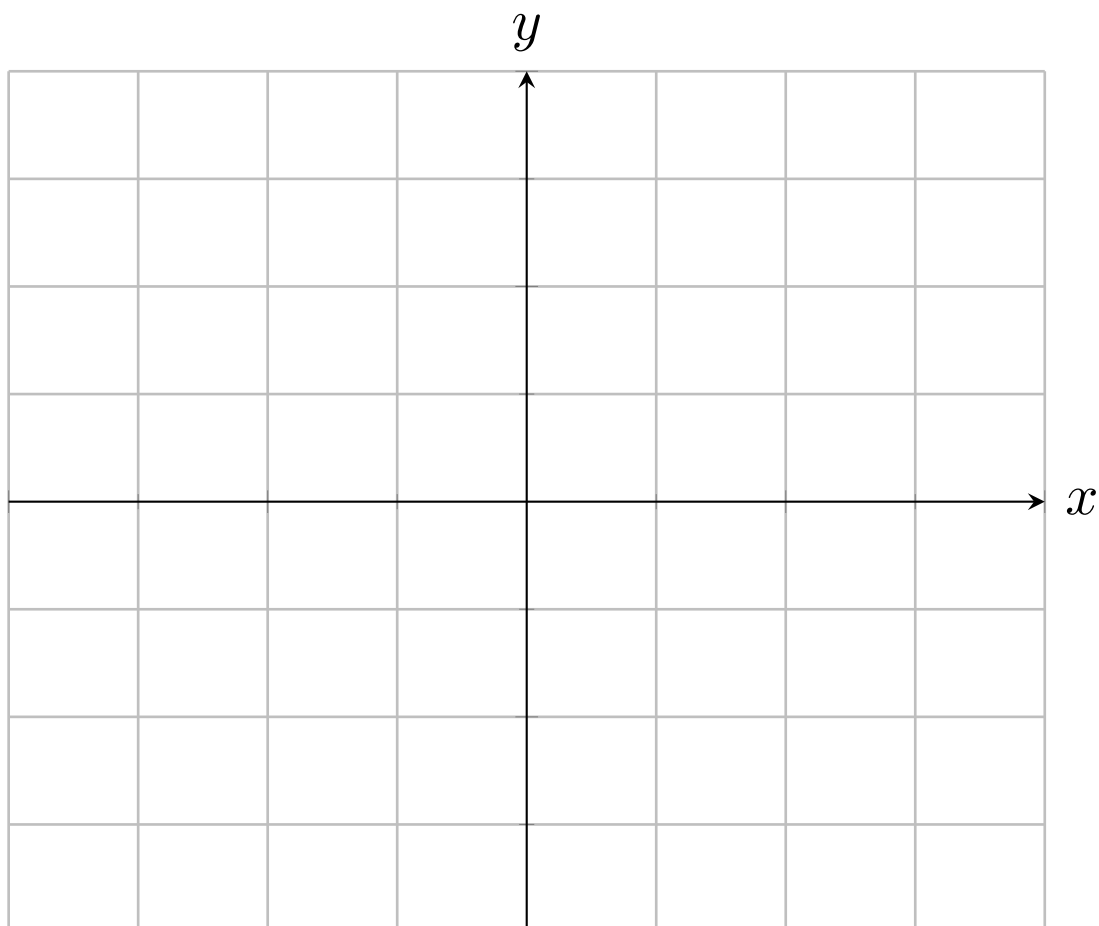
(d) (2 points) On which open interval(s) is f concave down? If there are none, write “none”.

(e) (2 points) Find the vertical asymptote(s) in the graph of f . If there are none, write “none”.

(f) (2 points) Find the horizontal asymptote(s) in the graph of f . If there are none, write “none”.

(Problem 10, continued. Recall that $f(x) = \frac{x+1}{2x+3}$, $f'(x) = \frac{1}{(2x+3)^2}$, $f''(x) = \frac{-4}{(2x+3)^3}$.)

(g) (3 points) Sketch the graph of f on the interval $-4 \leq x \leq 4$.



11. Calculate the following integrals. Be sure to simplify your answers.

(a) (5 points) $\int \frac{(x - \sqrt{x})^2}{x} dx$

(b) (5 points) $\int_0^{\sqrt{\pi}/2} x \cos(x^2) dx$

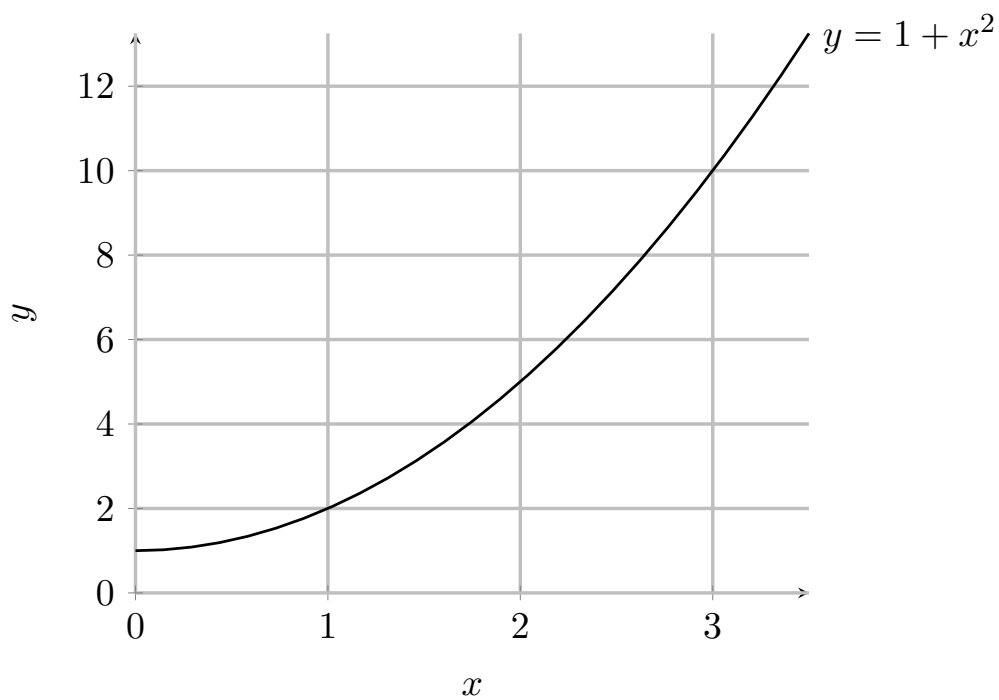
(c) (5 points) $\int \frac{2x + 3}{(x^2 + 3x + 4)^2} dx$

12. A sailboat is traveling across the Pacific ocean but the winds are shifting unfavorably, so the boat is accelerating at a rate of -2 kilometers/hour². Suppose that at time $t = 0$, the boat is 300 kilometers east of Honolulu and traveling east at a speed of 10 kilometers/hour.
- (a) (5 points) Write down a function that describes the position of the boat (in kilometers east of Honolulu) at time t (in hours).
- (b) (3 points) At what time t (in hours) will the boat reach an (eastward) speed of 4 kilometers/hour?
- (c) (4 points) At what time t (in hours) will the boat first reach a position of 324 kilometers east of Honolulu?

13. Consider the function $f(x) = 1 + x^2$.

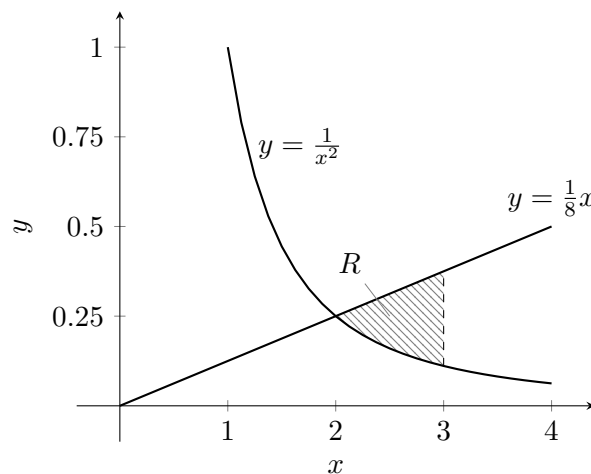
- (a) (4 points) Estimate $\int_0^3 f(x) dx$ with a Riemann sum using 3 subintervals of equal width and left endpoints.

- (b) (4 points) In the plot below, sketch the rectangles that you used in part (a).



- (c) (2 points) Which is larger: your estimate from part (a) or the exact value of $\int_0^3 f(x) dx$?

14. Let R be the region bounded by the curve $y = \frac{1}{x^2}$ and the lines $x = 3$ and $y = \frac{1}{8}x$, as shown below.



- (a) (5 points) Find the area of the region R .

- (b) (4 points) Set up **but do not evaluate** an integral for the volume of the solid obtained by rotating R about the x -axis.

- (c) (4 points) Set up **but do not evaluate** an integral for the volume of the solid obtained by rotating R about the y -axis.