Trigonometric identities

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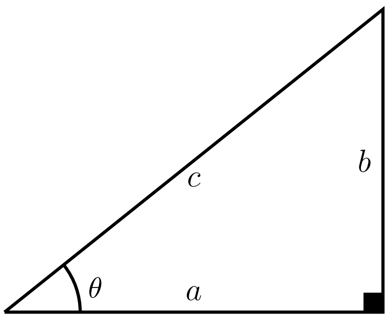
1 Trigonometry

1.1 Right angle triangle



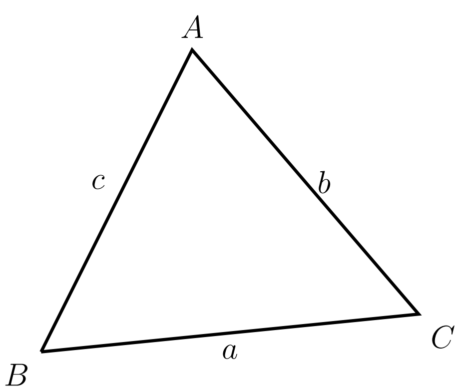
$$\sin\theta = \frac{\text{opposite side of }\theta}{\text{hypothenuse}} = \frac{b}{c} \qquad \csc\theta = \frac{\text{hypothenuse}}{\text{opposite side of }\theta} = \frac{c}{b}$$

$$\cos\theta = \frac{\text{adjacent side of }\theta}{\text{hypothenuse}} = \frac{a}{c} \qquad \sec\theta = \frac{\text{hypothenuse}}{\text{adjacent side of }\theta} = \frac{b}{c}$$

$$\tan\theta = \frac{\text{opposite side of }\theta}{\text{adjacent side of }\theta} = \frac{b}{a} \qquad \cot\theta = \frac{\text{adjacent side of }\theta}{\text{opposite side of }\theta} = \frac{a}{b}$$

Pythagore's formula : $a^2 + b^2 = c^2$
Remarks : $\tan x = \dfrac{\sin x}{\cos x} \quad \cot x = \dfrac{\cos x}{\sin x} \quad \sec x = \dfrac{1}{\cos x} \quad \csc x = \dfrac{1}{\sin x}$.

1.2 Abitrary triangle



Sinus Law

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Cosinus Law

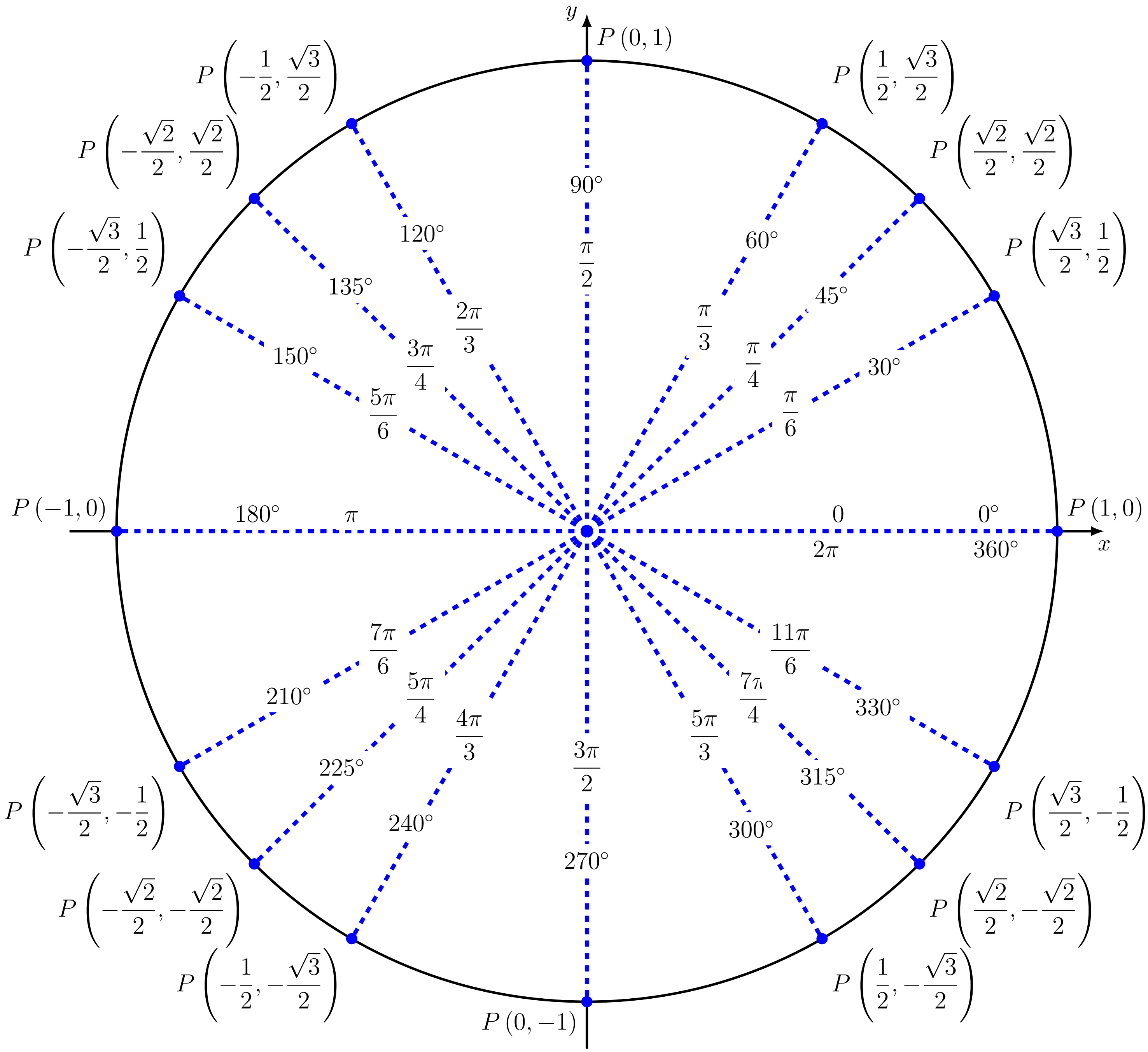
$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ac\cos C$$

Remark : If $\angle C = 90°$, $\angle B = 90°$ or $\angle C = 90°$, then we obtain the Pythagore's formula for the right-angle triangle by switching the roles of a, b, and c in the cosinus law.

1.3 Trigonometric circle



Conversion formula : We go from angles in degrees to angles in radians by using the following relation :

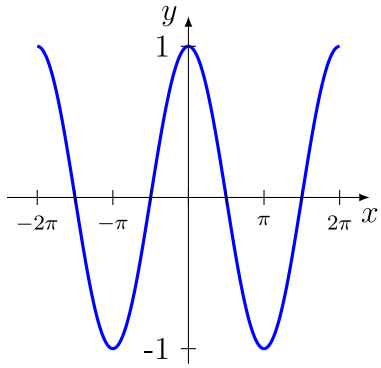
$$\frac{x \text{ rad.}}{\pi \text{ rad.}} = \frac{\theta^\circ}{180^\circ}.$$

1.4 Trigonometric identities

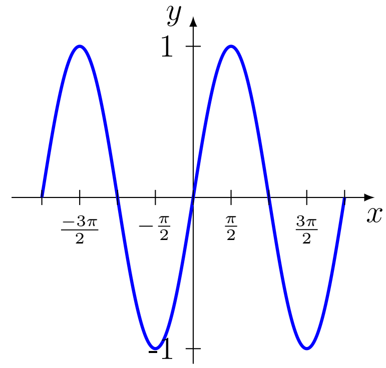
$\sin(-x) = -\sin x$	$\sin\left(\frac{\pi}{2} - x\right) = \cos x$	$\sin(\pi - x) = \sin x$
$\sin\left(x + \frac{\pi}{2}\right) = \cos x$	$\sin(x + \pi) = -\sin x$	$\sin(x + 2\pi) = \sin x$
$\cos(-x) = \cos(x)$	$\cos\left(\frac{\pi}{2} - x\right) = \sin x$	$\cos(\pi - x) = -\cos x$
$\cos\left(x + \frac{\pi}{2}\right) = -\sin(x)$	$\cos(x + \pi) = -\cos x$	$\cos(x + 2\pi) = \cos x$
$\tan(-x) = -\tan(x)$	$\tan\left(\frac{\pi}{2} - x\right) = \cot x$	$\tan(\pi - x) = -\tan x$
$\tan\left(x + \frac{\pi}{2}\right) = -\cot(x)$	$\tan(x + \pi) = \tan x$	$\tan(x + 2\pi) = \tan x$
$\cot(-x) = -\cot(x)$	$\cot\left(\frac{\pi}{2} - x\right) = \tan x$	$\cot(\pi - x) = -\cot x$
$\cot\left(x + \frac{\pi}{2}\right) = -\tan(x)$	$\cot(x + \pi) = \cot x$	$\cot(x + 2\pi) = \cot x$

$\cos^2 x + \sin^2 x = 1$	$\sin^2 x = \dfrac{1 - \cos 2x}{2}$
$\cos(2A) = \cos^2 A - \sin^2(A)$	$\cos^2 x = \dfrac{1 + \cos 2x}{2}$
$\sin(A + B) = \sin A \cos B + \cos A \sin B$	$\sin x \cos x = \dfrac{\sin 2x}{2}$
$\sin(A - B) = \sin A \cos B - \cos A \sin B$	$2 \sin x \cos y = \sin(x - y) + \sin(x + y)$
$\cos(A + B) = \cos A \cos B - \sin A \sin B$	$2 \sin x \sin y = \cos(x - y) - \cos(x + y)$
$\cos(A - B) = \cos A \cos B + \sin A \sin B$	$2 \cos x \cos y = \cos(x - y) + \cos(x + y)$
$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$	$\operatorname{Arc}\sin x + \operatorname{Arc}\cos x = \frac{\pi}{2}$
$\tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$	$\operatorname{Arc}\tan x + \operatorname{Arc}\tan \frac{1}{x} = \frac{\pi}{2}$ si $x > 0$
$\cot^2 x + 1 = \csc^2 x$	$\operatorname{Arc}\tan x + \operatorname{Arc}\tan \frac{1}{x} = -\frac{\pi}{2}$ si $x < 0$
$1 + \tan^2 x = \sec^2 x$	$\sin^3(x) = \dfrac{3 \sin x - \sin(3x)}{4}$
$\tan(2A) = \dfrac{2 \tan A}{1 - \tan^2 A}$	$\cos^3(x) = \dfrac{3 \cos x + \cos(3x)}{4}$

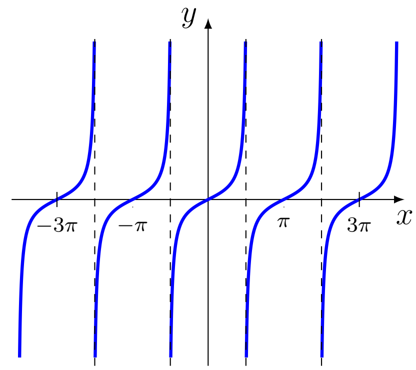
1.5 Graphs of the trigonometric functions



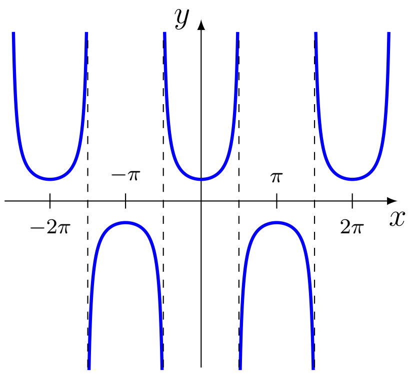
(a) $f(x) = \cos x$



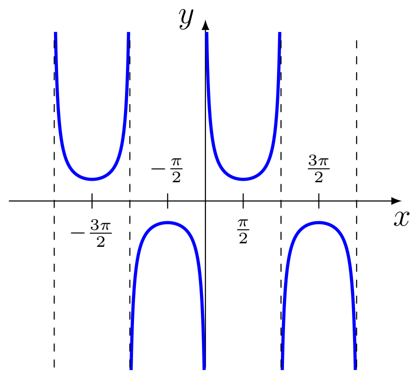
(b) $f(x) = \sin x$



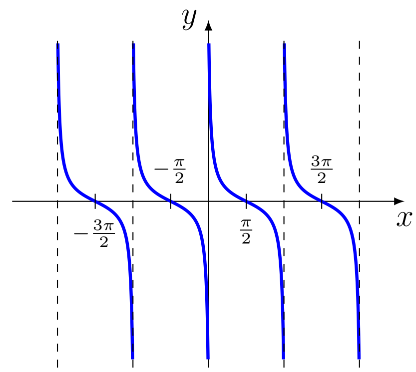
(c) $f(x) = \tan x$



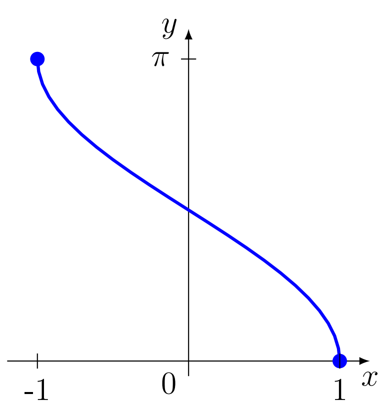
(d) $f(x) = \sec x$



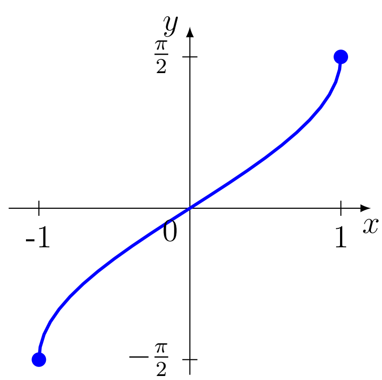
(e) $f(x) = \csc x$



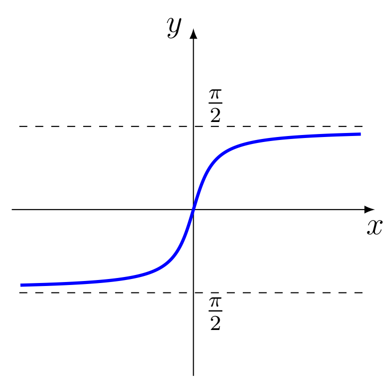
(f) $f(x) = \cot x$



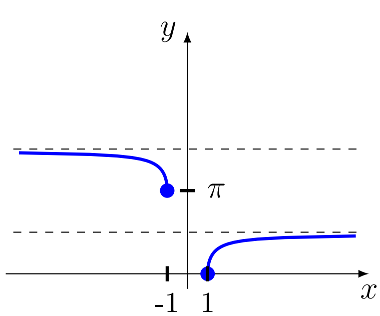
(g) $f(x) = \text{Arc}\cos(x)$



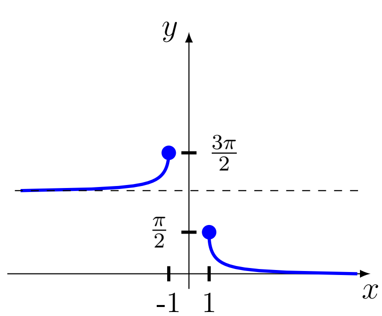
(h) $f(x) = \text{Arc}\sin(x)$



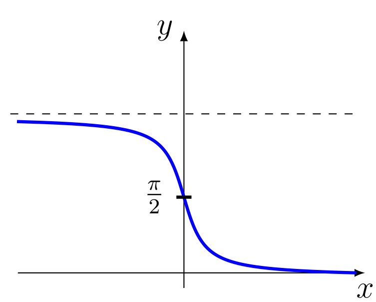
(i) $f(x) = \text{Arc}\tan(x)$



(j) $f(x) = \text{Arc}\sec(x)$



(k) $f(x) = \text{Arc}\csc(x)$



(l) $f(x) = \text{Arc}\cot(x)$