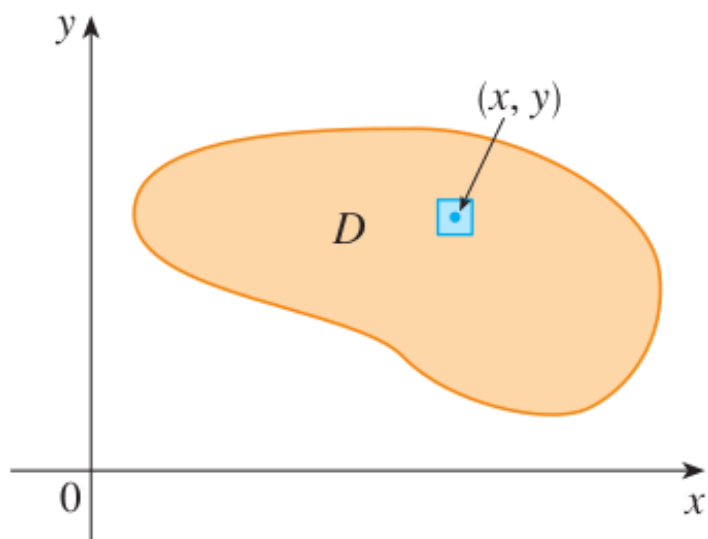# Chapter 15

Multiple Integrals

15.4 Applications of double integrals
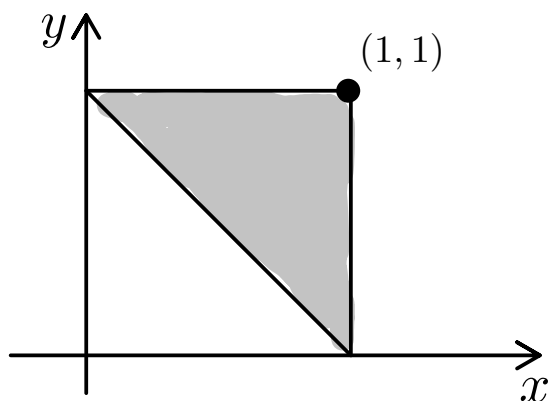
Density, mass and charge

$$m = \iint_D \rho(x, y) \, dA$$

$$Q = \iint_D \sigma(x, y) \, dA$$

EXAMPLE. Mass is distributed over the triangular region D below. The mass density at (x, y) is $\rho(x, y) = xy$, measured in kg/m^2. Find the total mass.

Moments and center of mass.

Moment about the x-axis

$$M_x = \iint_D y\rho(x,y)\, dA$$

Moment about the y-axis

$$M_y = \iint_D x\rho(x,y)\, dA$$

EXAMPLE. Find the moments about the x-axis and y-axis for the lamina from the previous example.

Center of mass $(\overline{x}, \overline{y})$

$$\overline{x} = \frac{M_y}{m} \qquad \text{and} \qquad \overline{y} = \frac{M_x}{m}$$

EXAMPLE. Find the center of mass for the lamina in the previous examples.

EXAMPLE 3 The density at any point on a semicircular lamina is proportional to the distance from the center of the circle. Find the center of mass of the lamina.

Moment of Inertia.

Inertia about the x-axis

$$I_x = \iint_D y^2 \rho(x, y)\, dA$$

Inertia about the y-axis

$$I_y = \iint_D x^2 \rho(x, y)\, dA$$

Inertia about the origin

$$I_0 = \iint_D (x^2 + y^2)\rho(x, y)\, dA$$