Problem 6

We first have that

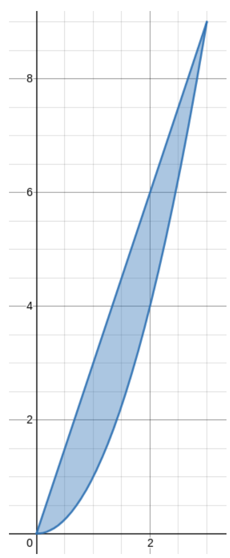
$$\int_0^{e^v} \sqrt{1+e^v}\, dw = \sqrt{1+e^v}\ (w)\big|_0^{e^v} = e^v\sqrt{1+e^v}.$$

So

$$\int_0^1 \int_0^{e^v} \sqrt{1+e^v}\, dw dv = \int_0^1 e^v\sqrt{1+e^v}\, dv = \frac{2}{3}((1+e)^{3/2} - 2\sqrt{2}) \approx 2.894.$$

Problem 14

The first thing to do is to draw the region D.



We see that the curves $y = x^2$ and $y = 3x$ intersects at the points $(0,0)$ and $(3,9)$.

Type I We have $0 \leq x \leq 3$ and $x^2 \leq y \leq 3x$. So the functions bounding the values of y are x^2 and $3x$. As a type I, the domain is written as

$$D = \{(x,y)\ :\ 0 \leq x \leq 2,\ x^2 \leq y \leq 3x\}.$$

Type II We see that $0 \leq y \leq 9$ and since $x \geq 0$, the curves bounding the values of x are $x = y/3$ and $x = \sqrt{y}$. As a a type II, the domain is written as

$$D = \{(x,y)\ :\ y/3 \leq x \leq \sqrt{y},\ 0 \leq y \leq 9\}.$$

Now the integral is

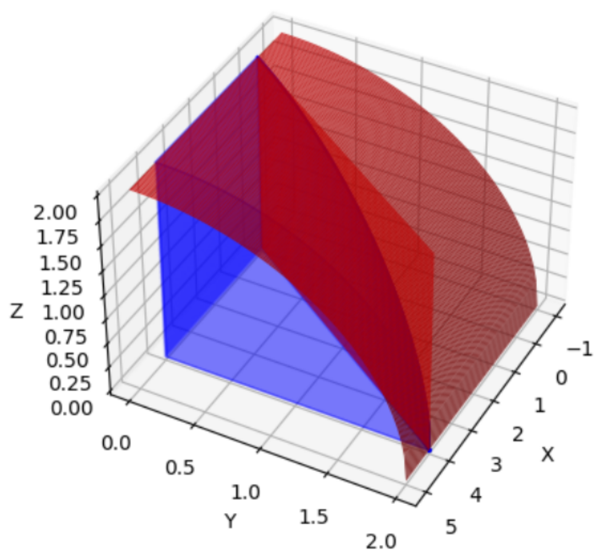
$$\int_0^3 \int_{x^2}^{3x} xy\,dydx = \int_0^3 x\left(\frac{9x^2 - x^4}{2}\right) dx = \int_0^3 \frac{9x^3 - x^5}{2} = \frac{243}{8}.$$

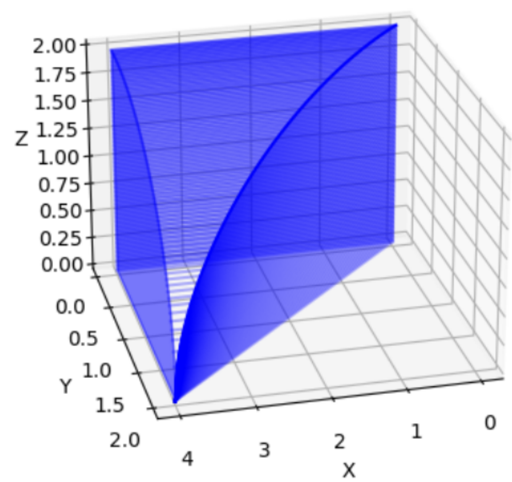
If you chose the other way, then your integral should look like this:

$$\int_0^9 \int_{y/3}^{\sqrt{y}} xy\,dxdy.$$

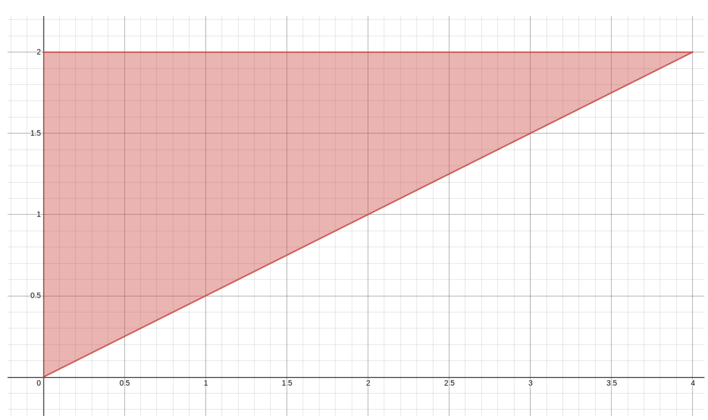
Problem 30

The solid we are trying to find the volume is represented in the figure below.





To find the domain of integration D, we have to project the surfaces $y^2 + z^2 = 4$ and $x = 2y$ on the XY-place. For the first surface, we obtain $y = \pm 2$ (two horizontal lines in the XY-plane) and $x = 2y$ (a line with slope $1/2$). So the domain of integration is the following region: So, the



domain D is

$$D = \{(x, y) \,:\, 0 \leq x \leq 2y,\, 0 \leq y \leq 2\}.$$

The function to integrate is $z = \sqrt{4-y^2}$. Thus, the volume of the solid S is given by

$$V(S) = \int_0^2 \int_0^2 y\sqrt{4-y^2}\, dxdy = \int_0^2 2y\sqrt{4-y^2}\, dy.$$

The integral with $2y\sqrt{4-y^2}\, dy$ is done by a change of variable and we get

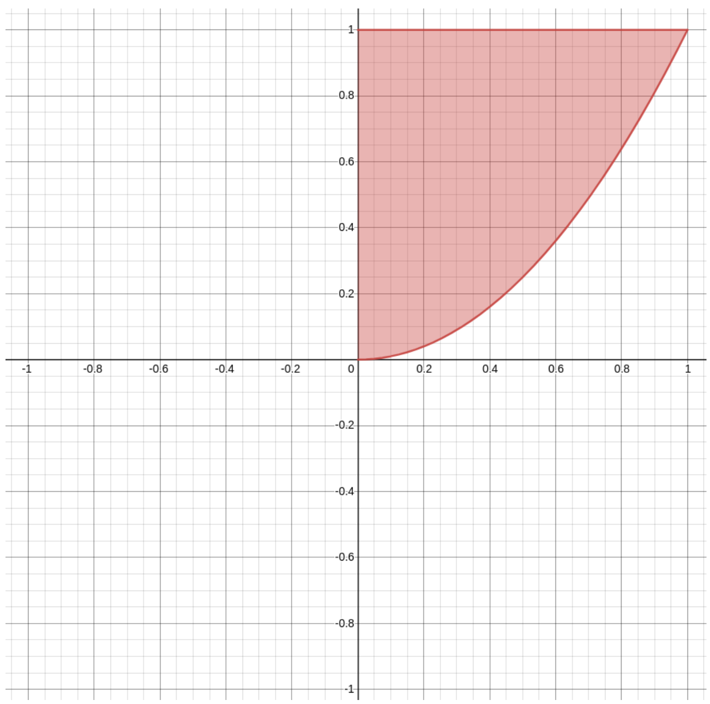
$$\int_0^2 2y\sqrt{4-y^2}\, dy = \frac{16}{3}.$$

Thus, the volume of the solid is

$$V(S) = \frac{16}{3}.$$

Problem 52

From the limits in the integrals, we see that $0 \le x \le 1$ and that $x^2 \le y \le 1$. So the region of integration looks like this: So the region D is the region bounded by the curves $x = 0$, $y = x^2$,



and $y = 1$. Since $x \ge 0$, the region D is also the region bounded by the curves $x = 0$, $x = \sqrt{y}$, and $y = 1$. So we can say that

$$D = \{(x, y) : 0 \le x \le \sqrt{y}, 0 \le y \le 1\}.$$

Thus, the integral now becomes

$$\int_0^1 \int_0^{\sqrt{y}} \sqrt{y}\sin y\, dxdy = \int_0^1 \sqrt{y}\sin(y)\Big(\sqrt{y} - 0\Big)\, dy = \int_0^1 y \sin y\, dy.$$

After an integration by parts, we get the value of the integral:

$$\int_0^1 \int_0^{\sqrt{y}} \sqrt{y}\sin y\, dxdy = \sin(1) - \cos(1) \approx 0.301168$$