

Problem 16

When $z = 0$, each $(x, y, 0)$ is mapped to $\mathbf{i} + 2\mathbf{j}$. So in the xy -plane, we should have the same vectors. This is exactly the plot I.

Problem 18

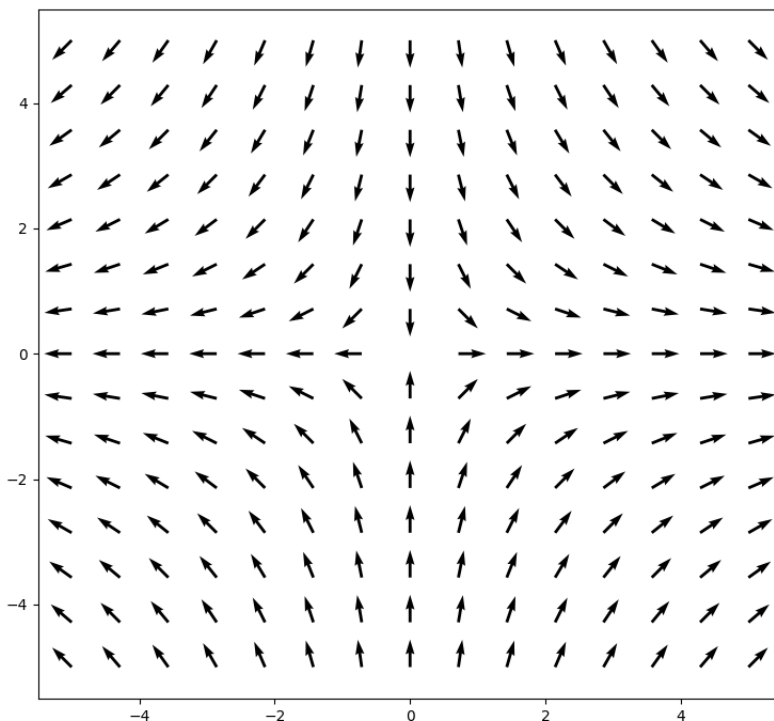
When x , y , and z are small, the vector $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ has a small length. If $x \neq 0$, $y \neq 0$, and $z \neq 0$, then the vector $x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ is pointing in the opposite direction of the origin (like a vector emanating from the origin, starting at the point (x, y, z)). Also, we see that if $x = y = z = 0$, then we obtain the zero vector. The only plot that has the zero vector is the plot II.

Problem 26

We have

$$f_x(x, y) = x \quad \text{and} \quad f_y(x, y) = -y.$$

So the gradient vector field is $\nabla f = \mathbf{F}(x, y) = x\mathbf{i} - y\mathbf{j}$. The picture below shows a plot of the vector field.



Problem 30

The function is $f(x) = x^2 + xy$. So its gradient is $\vec{\nabla}f(x, y) = (2x + y)\vec{i} + x\vec{j}$. The x -coordinate of the vector field is zero if $y = -2x$. In this case, the vector field looks like

$$\vec{\nabla}f(x, y) = -(y/2)\vec{j}.$$

Also, when $y > -2x$, then the x coordinate of the vector field is positive and all the vectors in the vector field must point to the east (to the right, in the same direction to the positive x -axis). When $y < -2x$, then the x coordinate of the vector field is negative and all the vectors in the vector field must point to the west (to the left, in the opposite direction to the positive x -axis).

So the corresponding representation is IV.

Problem 34

The vector at $(x, y) = (1, 3)$ is $\vec{F}(1, 3) = \langle 1, -1 \rangle$. So, the new position of the particle would be

$$\langle x_1, y_1 \rangle = (1, 3) + \Delta t \vec{F}(1, 3) = \langle 1, 2 \rangle + 0.05 \langle 1, -1 \rangle = \langle 1.05, 1.95 \rangle.$$