## L.1 Mathematical Statements

PROBLEM 1. Are the following sentences statements? If it is, say if it is true or false. If it is not, explain briefly why.

a) $|-12| = -12$.

b) $x < 0$.

c) Is that an odd integer?

d) If $a = 2$ and $b = 4$, then $a + b = 6$.

## L.2 Logic and Mathematical Language

PROBLEM 2. Give the converse and the contrapositive of the following conditional statements.

a) If it is Saturday, then Angela sleeps in.

b) If it rains outside, then I will use my umbrella.

c) If I went surfing, then the surf was bigger than 4 feet high.

PROBLEM 3. Write useful negations of the following statements in English. You can use symbols to simplify the statement.

a) It is raining and Charlie is cold.

b) If it is raining, then Charlie is cold.

c) For every real number $x$, there exists a real number $y$ such that $x + y = 0$.

d) $|a| > 0$ if and only if $a \neq 0$.

PROBLEM 4.

a) By constructing the truth table of $P \Rightarrow Q$ and $Q \Rightarrow P$, show when a conditional statement and its converse do not have the same truth values.

b) By constructing the truth table of $P \Rightarrow Q$ and $(\neg Q) \Rightarrow (\neg P)$, show a conditional statement and its contrapositive always have the same truth values.

## L.3 Methods of Proof

PROBLEM 5. Suppose that $a$ and $b$ are integers. Prove each of the following.

a) If $a$ and $b$ are both odd, then $a + b$ is even.

b) If $a$ is even and $b$ is odd, then $a + b$ is odd.

**Problem 6.** A rational number is a number $q$ that can be put in the form of a fraction, that is there exist two integers $n$ and $m$ such that $q = n/m$. Show that $\sqrt{2}$ is not rational.

**Problem 7.** Prove that there exist integers $m$ and $n$ such that $2m + 3n = 12$.

**Problem 8.** A sequence $(x_n)$ is a ordered list $(x_1, x_2, x_3, \ldots)$ of real numbers. We say that a sequence $(x_n)$ converges to a real number $a$ if $\forall \varepsilon > 0$, $\exists N \geq 0$ such that $\forall n \geq N$, $|x_n - a| < \varepsilon$.

a) Show that the sequence $(x_n)$ defined by $x_n = \frac{1}{n^2}$ converges to 0.

b) Show that the sequence $(x_n)$ defined by $x_n = \frac{n+1}{n}$ converges to 1.