SECTION 2.1 : Regions in the Complex planes

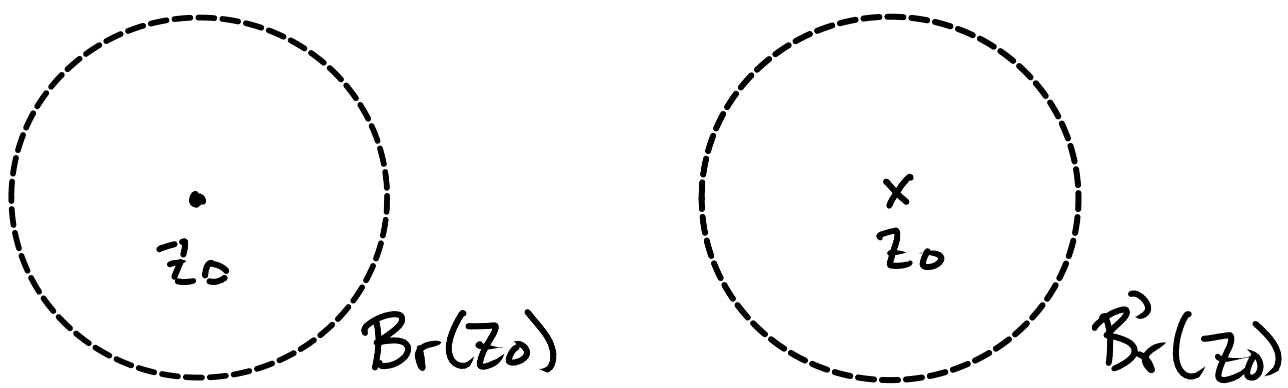
DEF. 2.1.1 (Neighborhood)

A neighborhood of a point $z_0 \in \mathbb{C}$ is the disk

$$B_r(z_0) = \{ z \in \mathbb{C} : |z - z_0| < r \}$$

A deleted neighborhood of $z_0 \in \mathbb{C}$ is the disk $B_r(z_0)$ from which we removed z_0:

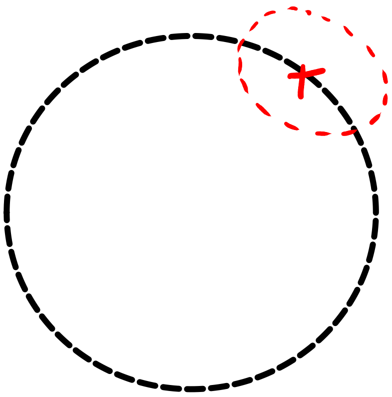
$$B_r'(z_0) = \{ z \in \mathbb{C} : 0 < |z - z_0| < r \}$$



Def. 2.1.2 Let $S \subset \mathbb{C}$.

A point $z_0 \in \mathbb{C}$ is a boundary point of S if $\forall r > 0$,

$$B_r(z_0) \cap S \neq \emptyset \text{ and } B_r(z_0) \cap S^c \neq \emptyset.$$



Boundary of a $B_r(z_0)$ is the circle

$$C_r(z_0) = \{z : |z - z_0| = r\}$$

DEF 2.1.4 (Open set)

A set $S \subset \mathbb{C}$ is open if for any $z_0 \in S$, $\exists r > 0$ such that $B_r(z_0) \subset S$.

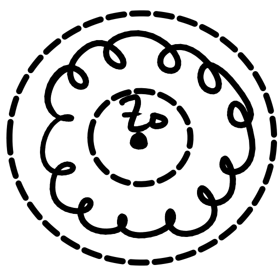
DEF. 2.1.5 (Closed set)

A set $S \subset \mathbb{C}$ is closed if S contains all of its boundary points.

Remark ∂S is the boundary of S, that is the set of boundary points of S. So, S is closed if $\partial S \subset S$.

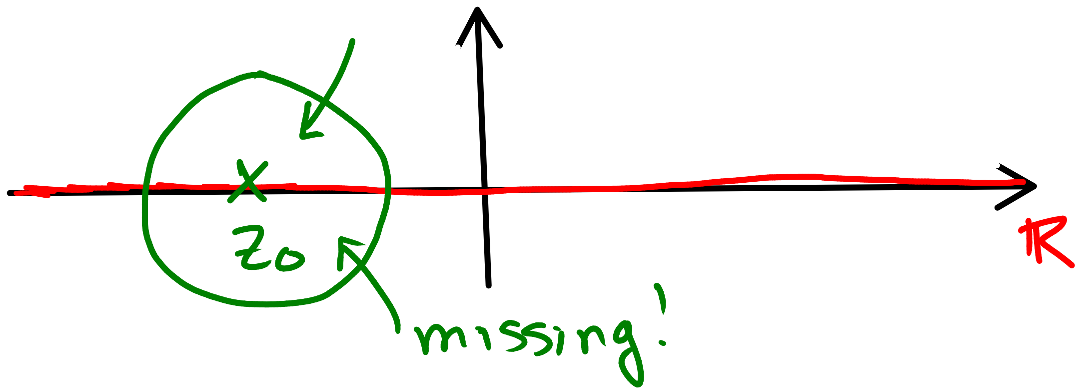
Examples (open)

- An disk $B_r(z_0)$ is open.
- The empty set $\emptyset$ is open.
- $\mathbb{C}$ is open.
- Annulus:



$r_2 < |z - z_0| < r_1$

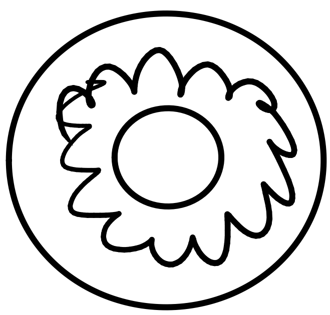
- Non-example: $\mathbb{R}$ is not open.



Examples (Closed)

- $$\overline{B_r(z_0)} = B_r(z_0) \cup \partial B_r(z_0)$$
  $$= \{ z : |z - z_0| \leq r \}$$

  is closed.
- $\emptyset$ is closed.
- $\mathbb{C}$ is closed.
- Closure Annulus:



$r_2 \leq |z| \leq r_1$

- $\mathbb{R}$ is closed.
- $\{z_0\}$ is closed.