Problem 16

Let $F(z) = \frac{1}{6}(z^2-1)^3$. Then this is analytic on $\Omega = \mathbb{C}$ and $F'(z) = \frac{3(z^2-1)^2(2z)}{6} = (z^2-1)^2 z$. Hence, from Theorem 3.3.4,

$$\int_{[z_1,z_2,z_3]} (z^2-1)^2 z\, dz = F(z_3) - F(z_1) = \tfrac{1}{6}(-2)^3 - \tfrac{1}{6}(-1)^3 = -\tfrac{7}{6}.$$

Problem 17

Let $F(z) = \frac{z^3}{3}$, analytic on $\Omega = \mathbb{C}$. We have

$$z_1 = e^{i(0)} + 3e^{2i(0)} = 4 \quad \text{and} \quad z_2 = e^{i(\pi/4)} + 3e^{2i(\pi/4)} = 1 + 4i.$$

Hence, by Theorem 3.3.4, we get

$$\int_\gamma z^2\, dz = \tfrac{(1+4i)^3}{3} - \tfrac{(4)^3}{3} = -\tfrac{47+52i}{3} - \frac{64}{3} = -37 - \tfrac{52}{3} i.$$

Problem 19

Notice that $F(z) = ze^z - e^z = (z-1)e^z$ is an antiderivative of ze^z and it is analytic on $\mathbb{C}$. By Theorem 3.3.4,

$$\int_{[z_1,z_2,z_3]} ze^z\, dz = (-1 - i\pi - 1)e^{-1-i\pi} - (\pi - 1)e^{\pi} \approx -48.82 + 1.16i.$$

Problem 22

Notice that $\sin^2(z) = \frac{1-\cos(2z)}{2}$. Hence,

$$\int_\gamma \sin^2(z)\, dz = \tfrac{1}{2} \int_\gamma 1 - \cos(2z)\, dz.$$

Notice that $F(z) = z - \frac{1}{2}\sin(2z)$ is an antiderivative of $1 - \cos(2z)$. Since γ is a closed path, we get

$$\int_\gamma 1 - \cos(2z)\, dz = 0$$

and hence the integral is 0.